Wage Contracts and Financial Frictions

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Job Market Paper

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January 26, 2025

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Abstract

Financial crises often lead to drastic reductions in firms' access to credit, impacting significantly their ability to finance their operations. This paper shows that firms can partly offset the effects of these shocks by optimally adjusting their wage bills. We augment a baseline financial frictions model to account for two well-documented features of the labor market: wages are set at the firm level and within long-term employment relationships. Because of these features, wage dynamics depend on the financial conditions of firms, reflecting a trade-off between smoothing wages of risk-averse workers and investing in capital. We validate the model predictions on wage dynamics using matched employer-employee data from Italy. We find that more constrained firms adjust wages more in response to idiosyncratic shocks. In addition, firms that suffer the most during recessions backload wages by paying workers relatively more in the future than today. When matching these statistics with our general equilibrium model, we find that these wage adjustments reduce the sensitivity of output to financial shocks by 20%: wage backloading enhances investment and job creation while improving allocative efficiency. We conclude by studying policies aimed at reducing inputs cost during recessions. Our findings show that these wage adjustments diminish the effectiveness of temporary payroll subsidies while enhancing the effectiveness of temporary investment subsidies in stimulating output.

We are sincerely grateful to Adrien Auclert, Luigi Bocola, Patrick Kehoe, Elena Pastorino and Christopher Tonetti for their advice and invaluable support. We would also like to thank Rafael Berriel, Nick Cao, Daniele Caratelli, Federico Cingano, Sebastian Di Tella, John Grigsby, Pete Klenow, Chad Jones, Virgiliu Midrigan, Sauro Mocetti, Luigi Pistaferri, Enrico Sette and seminar participants at Stanford University, Federal Reserve Bank of Chicago, Federal Reserve Bank of Minneapolis for feedbacks and comments. We are grateful to Francesco Beraldi for insightful discussions. This project was supported by the Gale and Steve Kohlhagen Fellowship in Economics through a grant to the Stanford Institute for Economic Policy Research. The opinions expressed in this paper are those of the authors alone and do not necessarily reflect the views of the Bank of Italy. All remaining errors are our own.

1 Introduction

After the Great Recession, there has been a wide consensus that tightening credit conditions played an important role in recent cycles. Yet, how much of the overall decline in output was actually due to the credit crunch remains an open question.

The central framework for understanding the macroeconomic effects of an economy-wide tightening of credit conditions (i.e., a credit crunch) hinges on the role of firms' credit, building on the work of Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999). The main mechanism is based on the idea that a reduction in the availability of credit forces employers to cut investment and hiring because of the shortage of funds, leading to large output losses from a credit crunch.

The standard mechanism implicitly relies on firms purchasing input in spot markets. In other words, whenever firms experience a financial shock, the burden of adjustment must fall on quantities because firms take input prices as given. This assumption is quantitatively important in explaining the large macroeconomic effects of financial shocks, especially when applied to the labor market, as the wage bill constitutes a significant share of firms' costs.

However, this assumption conflicts with two well-documented characteristics of the labor market: a significant portion of wages is determined at the firm level (Card, Heining, and Kline, 2013; Card et al., 2018), and employment relationships are typically long-term in nature. This last feature is crucial to capture how firms can adjust the wage growth of their workers over time. In fact, these characteristics of the labor market are consistent with evidence that firms adjust the wage growth of their workers over time and in response to shocks (Guiso, Pistaferri, and Schivardi, 2005, 2012), suggesting that firms can adjust wages rather than quantities when access to credit is limited.[1] How do firms adjust wage growth during periods of credit tightening? What are the macroeconomic implications of these wage adjustments? And how effective are stabilization policies aimed at reducing the cost of inputs during financial crises?

This paper proposes answers to these questions in four dimensions. First, we propose a general equilibrium model of frictional financial and labor markets with aggregate shocks, where wages are set at the firm level as part of long-term employment relationships, and firms face occasionally binding financial constraints. Second, we provide novel empirical evidence using matched employer-employee data that supports model predictions on wage dynamics: firms adjust wage growth over time and in response to shocks based on their financial conditions and to ease the effects of credit constraints. Third, we show that these wage adjustments are quantitatively important during the business cycle, as they

[1] Guiso, Pistaferri, and Schivardi (2005) finds that firms adjust wages in response to shocks in a way to partially insure workers' earnings from idiosyncratic fluctuations in firms' productivity. Guiso, Pistaferri, and Schivardi (2012) finds that firms in less financially developed regions pay a steeper wage-tenure profile, thus adjusting wage payments over time.

significantly mitigate the output loss from a credit crunch. Fourth, we study the effects of temporary payroll and investment subsidies, commonly used stabilization policies to reduce the cost of inputs. We find that optimal wage adjustments over long-term employment relationships reduce the effectiveness of payroll subsidies in stimulating output, but enhance the effectiveness of investment subsidies.

We build on a canonical model of firms' dynamics under financial frictions (Moll, 2014; Khan and Thomas, 2013). In our economy, firms face idiosyncratic productivity shocks and issue debt in order to finance their operations subject to a potentially binding collateral constraint. As in Jermann and Quadrini (2012), we model an aggregate financial shock as a tightening of this financial constraint. Importantly, we introduce search frictions in the labor market and long-term wage contracts between firms and workers in the tradition of Thomas and Worrall (1988).

Credit market frictions generate a trade-off between providing insurance to risk-averse workers and investing in capital. To understand the mechanism, consider a firm that currently operates below "optimal scale" due to binding borrowing constraints. Over time, as this constraint is gradually eased, the firm's output will increase.[2] Risk-averse workers would like to receive a constant wage throughout this transition. However, when credit constraints are tight, the firm would like to pay higher wages in the future relative to today. In other words, firms would like to temporarily backload wages to implicitly borrow from their workers. More generally, wages adjust over time and in response to shocks depending on firm-specific financial conditions.

In the model, we illustrate two key findings. First, firms significantly affected by financial frictions adjust wages more in response to firms' specific idiosyncratic shocks, as it is more costly for these firms to hedge workers against idiosyncratic shocks. Second, firms temporarily backload wage payments when their financial constraints bind. This means that during a credit crunch, firms more impacted by credit tightening can reduce the cost of labor to invest more in productive capital and ease the credit constraint, with the implicit promise of future wage increases.

We propose new empirical evidence supporting these two key predictions of the model on how wage dynamics depends on firms' financial conditions. We use matched employer-employee data from Italy, including administrative data on workers' compensation and firms' balance sheets. First, using a model-consistent indicator alongside several others, we show that in the cross section firms that are more financially constrained adjust wages significantly more in response to idiosyncratic shocks. Specifically, we estimate the "pass-through" of value added per worker to wages, a commonly used statistic that measures the extent to which workers are subject to firm-specific shock. We find that this pass-through coefficient is almost twice as large for more financially constrained firms.

[2]This dynamics is standard and common to several models of financial frictions, as in Moll (2014) and Midrigan and Xu (2014).

Second, we show that in recession, more financially constrained firms backload wages of new hires by paying their workers according to a temporarily steeper wage-tenure profile. We compare the wage-tenure profile of workers hired during the Great Recession by more and less constrained firms, and we find that wages grow approximately 2 percentage points and 4 percentage points more after one and four years of tenure at more constrained firms.

The estimated model is consistent with both empirical evidence on heterogeneous wage dynamics across firms and stylized facts on aggregate wage dynamics. Despite the fact that firms adjust wage growth to alleviate the effects of a credit tightening, the average wage remains relatively stable over the cycle, consistent with evidence that it moves little during recessions (Grigsby, 2022). However, the modest cyclicality of the average wage masks substantial heterogeneity in the cross section, where financially constrained firms contract and substantially reduce wage growth, while unconstrained firms expand and moderately increase wage growth. In the aggregate, this firm-level heterogeneity in wage adjustments implies that the skewness of the wage adjustment distribution is lower in recessions than in booms, consistent with evidence documented by Adamopoulou et al. (2016). Crucially, since wage growth is positive on the corse of an employment relationship (Rubinstein and Weiss, 2006; Buchinsky et al., 2010), the mechanism at the core of this paper is not based on wage cuts for incumbent workers but rather on slower wage growth.

What are the macroeconomic implications of these wage adjustments? We use our model to answer this question. When firms temporarily backload wage payments during recessions, this frees resources for current *and* future investment. In fact, because the effects of financial frictions are persistent (Moll, 2014), an increase in current investment also increases future retained earnings and future investment. The effect of wage backloading on current *and* future investment enhances job creation by substantially increasing the surplus of hiring a worker. Additionally, the ability to pay state-contingent wages lowers the expected cost of hiring a worker by minimizing the expected present discounted value of all future wage payments, discounted using the firm-specific stochastic discount factor.

The quantitative analysis shows that firms' ability to adjust wage growth over long-term employment relationships substantially reduce the output loss from a credit crunch. Toward that purpose, we construct an alternative economy in which firms cannot commit to future wages, a restriction that prevents firms from adjusting wages over time and in response to shocks. This second economy works as the canonical model of financial frictions, so the comparison with the baseline will help to understand how far dynamic wage contracts go in smoothing the effects of shocks.[3] By comparing impulse response

[3]As in the canonical model of financial frictions, the allocative wage for job creation is the wage at the time of hiring. Therefore, any change in wages at time t must be accompanied by a change in employment at t. On the other hand, a key feature of our model is that firms can change wages at time t without affecting the labor supply decision of workers and therefore employment, as long as future wages also adjust to deliver the same present discounted value to workers.

functions to an aggregate financial shock in the two economies, we find that output drops by 20% less in our model.

The differential response of output to an aggregate financial shock is primarily driven by differences in aggregate employment and allocative efficiency. In the baseline economy, employment and investment fall less because dynamic wage contracts boost job creation and free resources for current investment. Although general equilibrium effects limit differences in aggregate investment, the baseline model reallocates capital towards more productive firms. As a result, the decline in total factor productivity, an indicator of allocative efficiency, is less severe with dynamic wage contracts.

Finally, we illustrate that incorporating the dynamic structure of wage contracts during financial crises has important policy implications. We study the impact of policies aimed at reducing input cost during recessions, such as temporary investment and payroll subsidies.[4] We find that a payroll subsidy on new hires is not as effective as a standard model would suggest because firms' optimal wage adjustments and payroll subsidies act as subsidies, meaning they are both aimed at reducing the cost of labor. As a result, payroll subsidies are less effective in stimulating output during recessions when firms optimally backload wages to reduce the cost of labor. On the other hand, we find that an investment subsidy is more effective than in a standard model because firms' optimal wage adjustments and investment subsidies act as complements. Indeed, when investment subsidies transfer resources to firms and make investment opportunities more attractive, financially constrained firms backload wages even more to free additional resources for investment, thus amplifying the stimulative effect of the policy. We illustrate these results quantitatively by simulating in the model investment and payroll subsidies similar to those implemented in the United States after the Great Recession.

Related Literature

This paper relates to a large literature that studies the role of financial frictions during recessions. Early research showed that financial imperfections can amplify shocks originated outside of the financial sectors, as in the work of Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999), Gertler and Karadi (2011), Arellano, Bai, and Kehoe (2019). Subsequent studies showed that shocks that originated in the financial sector can propagate to the rest of the economy leading to financial recessions, as in the work of Jermann and Quadrini (2012), Khan and Thomas (2013), Buera and Moll (2015)[5].

Assessing the quantitative importance of these mechanisms has been a large and ongoing area of research, as several forces can dampen or amplify their effects. For

[4]These policies have been implemented by several OECD countries, often as a temporarily accelerated tax depreciation and temporary payroll tax cuts on new hires.

[5]Other related papers studying the effects of shocks originated in the financial sector are Martín (2004), Vanasco and Asriyan (2014), Kiyotaki and Moore (2019).

example, Chari (2012) and Moll (2014) pointed out that non-financial firms might be able to self-finance themselves. More closely related to our paper, Di Tella (2017), Carlstrom, Fuerst, and Paustian (2016), Dmitriev and Hoddenbagh (2017), Asriyan, Laeven, and Martín (2021), Asriyan et al. (2024), study the link between optimal debt contracts and financial amplification, while Bocola and Bornstein (2023) study how trade credit within long-term supplier relationships amplifies the effects of financial shocks.[6]

Our paper differs from these studies by highlighting a new channel that mitigates the effects of financial shocks. In practice, we focus on wage contracts rather than financial contracts and provide rich empirical evidence from administrative data supporting the main mechanism. Conceptually, we emphasize the importance of contracts that are long-term in nature and contingent on idiosyncratic characteristics –and not only on aggregate variables.

This paper also links to the literature on optimal wage contracts within long-term employment relationships. Building on the seminal work of Thomas and Worrall (1988) and Harris and Holmstrom (1982), optimal wage contracts have been studied in the context of rich search models of the labor market by Burdett and Coles (2003), Shi (2009), Menzio and Shi (2010), Schaal (2017), Fukui (2020), Balke and Lamadon (2022), Souchier (2023), with an emphasis on the role of on-the-job search. However, this literature abstracts from firms' investment decisions and financial market imperfections limiting firms' access to credit. We see our paper as complementary to their work, as we study the role of financial frictions and firms' investment decision in affecting optimal wage contracts. Related to the seminal studies by Michelacci and Quadrini (2009) and Berk, Stanton, and Zechner (2010), which explore properties of optimal wage contracts with firms' financial constraints, we develop a business cycle model where wages respond to both idiosyncratic and aggregate shocks, provide empirical evidence supporting the model's predictions, and quantitatively assess their macroeconomic implications for business cycle and stabilization policies.

This paper also addresses the recent macroeconomic literature exploring the link between wage rigidity and financial frictions. The work of Favilukis, Lin, and Zhao (2020), Schoefer (2021), Wang (2022), Donangelo et al. (2019), and Acabbi, Panetti, and Sforza (2020), is based on the idea of "labor as leverage", where wage rigidity increases the leverage of firms, as poorly flexible payrolls act like predetermined debt obligations. Our research adds to this by considering wage rigidity arising endogenously from the optimal contract between firms and risk-averse workers. While nesting the idea of "labor as leverage" in our framework, we demonstrate through both modeling and data that financially constrained firms tend to adjust wages more after a shock. Crucially, we highlight the role of wage backloading arising within a long-term employment relationship in easing

[6]Other related papers introducing optimal contracts in business cycle models are Boldrin and Horvath (1995), Kehoe and Perri (2002), Cooley, Marimon, and Quadrini (2004).

financial constraints. Thus, we show that the view of long-term employment relationship as simply a source of wage rigidity misses important patterns in the data that have large quantitative implications over the business cycle.

Finally, this paper relates to several studies documenting the effectiveness of payroll and investment subsidies. Cahuc, Carcillo, and Le Barbanchon (2018) and House and Shapiro (2008) found that the stimulus of these policies is substantial, while Neumark and Grijalva (2017) and Zwick and Mahon (2017) highlighted their increased effectiveness during the Great Recession.[7] Using a general equilibrium model, we document rich interactions between these policies and how firms set wages over long-term employment relationships during a financial crisis. We show both conceptually and quantitatively that optimal wage adjustments implied by the dynamic nature of wage contracts make temporary payroll subsidies less effective at stimulating output while making temporary investment subsidies more effective.

The paper is organized as follows. Section 2 describes the model. Section 3 illustrates the model mechanism: it characterizes properties of dynamic wage contracts, it shows how wages vary with firms' financial conditions, and illustrates how hiring and investment decisions depend on the structure of wage contracts. Section 4 illustrates novel empirical evidence on wage dynamics that validates the predictions of the model. Section 5 presents the main quantitative results, discussing the macroeconomic implications of dynamic wage contracts for business cycle and stabilization policies. Section 6 concludes.

2 Model

We consider an economy populated by a continuum of entrepreneurs and workers. Entrepreneurs are heterogeneous in their productivity and produce output using capital and labor. There are financial frictions in the form of a collateral constraint, that is, the borrowing capacity of entrepreneurs is limited by a fraction of the value of their capital stock, which serves as collateral. We model aggregate financial shocks as a decrease in the collateral value of capital. These features of our environment are common to several business cycle models with financial frictions and heterogeneity, as Khan and Thomas (2013), Buera and Moll (2015), Kiyotaki and Moore (2019).

Workers and entrepreneurs meet in a frictional labor market and engage in long-term employment relationships. As a result, workers can be employed or nonemployed, and entrepreneurs can be either matched with workers or vacant. Before matching with a worker, entrepreneurs offer wage contracts that specify the path of wages for any possible history of future shocks, as in Thomas and Worrall (1988). We describe the environment in detail in Section 2.1, we illustrate the decision problems and value functions of en-

[7] Saez, Schoefer, and Seim (2019) also documented larger impacts of payroll subsidies on financially constrained firms

trepreneurs and workers in Section 2.2, we define macroeconomic aggregates in Section 2.3, we define the equilibrium in Section 2.4, and we conclude by discussing some of the model assumptions in 2.5

2.1 Environment

Time is discrete and indexed by $t = 0, 1, \ldots$. There is a continuum of entrepreneurs with measure 1, indexed by $j \in [0, 1]$, and a continuum of workers with measure M, indexed by $i \in [0, M]$. All agents in the economy have time-separable preferences with discount factor β, but entrepreneurs and workers differ in their utility functions. Entrepreneurs have utility

$$E_0 \sum_{t=0}^{\infty} \beta^t v\left(c_{jt}\right), \quad v(c) = \frac{c^{1-\sigma_E}}{1-\sigma_E}$$

and workers have utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{it}\right), \quad u(c) = \frac{c^{1-\sigma_W}}{1-\sigma_W}$$

Production technology

If entrepreneurs are not matched with a worker, they produce the value of home production $\bar{b}$. Similarly, non-employed workers also produce $\bar{b}$. Matched entrepreneurs produce output using a standard constant returns to scale production function $f(k, \ell)$, and they are heterogeneous in their idiosyncratic productivity z, so that output y_{jt} is given by

$$y_{jt} = z_{jt} \times f(k_{jt}, \ell_{jt})$$

Idiosyncratic productivity z follows a discrete Markov process, taking values $z \in \{z_1, \ldots, z_{N_z}\}$, with transition matrix Π_z. We assume that realizations of productivity shocks are independent across entrepreneurs and also independent over time. These assumptions imply a law of large numbers so the share of entrepreneurs experiencing any particular sequence of shocks is deterministic. As we assume that entrepreneurs can hire only one worker, we write the production function more compactly as $f(k) = f(k, 1)$. Entrepreneurs have access to a technology that can convert final good into physical capital one for one. In what follows we drop the j subscript whenever it is not needed for clarity.

Financial markets

Entrepreneurs are the only agents in the economy that have access to financial markets. They can borrow or save using uncontingent risk-free bonds that are in zero net-supply.

Borrowing is subject to a standard collateral constraint, as entrepreneurs can borrow an amount b_{t+1} that must be less or equal to a share ξ_t of their capital stock k_{t+1}, that is

$$b_{t+1} \leq \xi_t k_{t+1}$$

While the collateral constraint is assumed to be exogenous here, it can be obtained as an endogenous outcome in an environment with limited enforcement of debt contract, where entrepreneurs can decide to not repay their debt and steal profits and share $(1 - \xi_t)$ of their capital stock.[8]

The collateral value of capital ξ_t is common to all entrepreneurs and it follows a discrete Markov Process. It can take two values $\xi \in \{\xi_L, \xi_H\}$ with transition matrix Π_ξ. We interpret ξ_H as the collateral value of capital in normal times, and ξ_L as the collateral value in recession. The stochastic behaviour of the collateral value of capital allows us to study the macroeconomics implications of aggregate financial shocks, as in Jermann and Quadrini (2012), Khan and Thomas (2013), Buera and Moll (2015), Bocola and Bornstein (2023). We refer to a change from ξ_H to ξ_L as a financial shock, as when $\xi = \xi_L$ entrepreneurs face more limited access to credit.

We assume that workers do not have access to financial markets, that is they are hand-to-mouth. This is a common assumption in models of dynamic wage contracts as it simplifies the contracting problem, and is also in line with the view that firms have better access to financial markets than workers. We see this as a conservative assumption, as it greatly limits the ability of firms to adjust wages.

Labor market

Entrepreneurs can be matched to a worker or they can be vacant. Workers can be employed or not employed. Matched entrepreneurs and employed workers separate with probability ϕ. Non-employed workers and unmatched entrepreneurs meet in a frictional labor market with directed search, as in Moen (1997). In the economy there is a continuum of sub-markets with a constant returns to scale matching function $m(v, s)$, where v is the measure of unmatched entrepreneurs and s is the measure of workers searching in a given sub-market.

Each unmatched entrepreneur can open a vacancy in one sub-market. When an entrepreneur opens a vacancy he commits to a wage contract $\mathcal{C} = \{w_\tau(z^\tau, \xi^\tau)\}_{\tau=t}^{\infty}$ that specifies wages contingent on all future histories of idiosyncratic shocks z^τ and aggregate shocks ξ^τ. We assume that workers can also commit to a wage contract upon matching with an entrepreneur. Each non employed worker can search for a job in one sub-market. We use $\theta = v/s$ to denote the labor market tightness of each sub-market. Given the

[8] See Bocola and Lorenzoni (2023) for an example of the limited enforcement problem with a collateral constraint.

matching function, one can define the job finding probability $\lambda_w(\theta)$ and the probability of filling a vacancy $\lambda_f(\theta)$ as

$$\lambda_w(\theta) = \frac{m(v,s)}{s}, \qquad \lambda_f(\theta) = \frac{m(v,s)}{v}$$

Each sub-market is indexed by the tuple (θ, W), where W is the expected utility of a worker conditional on finding a job in that sub-market. When entrepreneurs open a vacancy in a sub-market indexed by (θ, W), they commit to a wage contract that will deliver the worker an expected utility equal to W. As each entrepreneur can open only one vacancy, we assume that there are no vacancy posting costs. A non employed worker can search for a job, but search is costly and it implies a disutility cost, as in models of non-participation similar to Krusell et al. (2017). We assume that non-employed workers who search for a job have to forgo a share x of the value of home production, in line with empirical evidence that non employed workers have to spend time searching for a job. Therefore, the flow utility of a non employed worker earning the flow output of home production $\bar{b}$ and searching for a job is equal to $u((1-x)\bar{b})$.

Timing

At the beginning of each period idiosyncratic and aggregate shocks z_{jt}, ξ_t are realized. Each period can be divided into two stages, that we label as morning (or *before* matching and separation) and afternoon (or *after* matching and separation).

In the morning, matched entrepreneurs produce output and pay wages to the employed workers. Unmatched entrepreneurs and non-employed workers produce $\bar{b}$. Then, unmatched entrepreneurs post vacancies and non-employed workers search for jobs. Unmatched entrepreneurs decide whether they want to open a vacancy or not, and if they do so they decide in which market indexed by (θ, W). Some entrepreneurs may not find it profitable to open a vacancy, and if so they will stay vacant until the beginning of the subsequent period. Similarly, non employed workers decide whether to search for a job or not. Conditional on searching, non employed workers choose a market indexed by (θ, W) where to locate.

At the end of each morning matching and separation take place. Matched entrepreneurs can become vacant with probability ϕ, while unmatched entrepreneurs who opened a vacancy in a market with tightness θ will be matched to a worker with probability $\lambda_f(\theta)$. Similarly, non-employed workers who search for a job in a market with tightness θ will be matched to an entrepreneur with probability $\lambda_w(\theta)$.

In the afternoon all agents consume. Workers consume the income earned in the morning, before matching and separation: if they were employed they consume the wage they earned, if they were not employed they consume $\bar{b}$ or $(1-x)\bar{b}$, depending on whether they searched for a job. All entrepreneurs solve a consumption/saving problem. We assume that unmatched entrepreneurs cannot hold capital, so they save using risk-free bonds.[9] Matched entrepreneurs decide on how much to borrow or save in the risk-free bonds, and how much capital stock to hold next period.[10]

2.2 Value functions and wage determination

We describe the problem of each agent recursively. First we discuss the recursive state space, and then we describe in details the maximization problem of each agent.

Recursive state space

We characterize the optimal contract recursively. We define a recursive contract as wages and promised utility $w'(z',\xi'), W'(z',\xi')$ that depends on current state variables and are contingent only on the realizations of shocks next period (z',ξ'). This formulation requires to include the utility promised to the worker W as a state variable in the problem of matched entrepreneurs. In other words, in each period, matched entrepreneurs choose state-contingent wages and promised utility for the next period, subject to a promise-keeping constraint where the expected utility of the worker must equal the utility W promised in the previous period.

Heterogeneity across matched entrepreneurs can be summarized by the exogenous state variable z and two endogenous state variables (m, W), where W is the utility promised to the worker and m is net worth, or cash-on hand, that is equal to the sum of output and undepreciated capital stock, minus wage payments and the repayment of outstanding debt, according to the law of motion:

$$m'(z',\xi') \leq z' f(k') + (1-\delta)k' - w'(z',\xi') - b' \tag{1}$$

Similarly, heterogeneity across unmatched entrepreneurs can be summarized by the exogenous state variable z and the endogenous state variable m. Heterogeneity across employed workers is fully summarized by their expected utility W, and there is no heterogeneity across non-employed workers as they are hand-to-mouth and there is no ex-ante heterogeneity. The aggregate state of the economy, that is denoted by S is summarized by the realization of the aggregate shock ξ, and distribution of matched and unmatched entrepreneurs over their states, that we denote by $\Lambda^m(m, W, z)$ and $\Lambda^v(m, z)$.

[9] Note that unmatched entrepreneurs would never choose to hold physical capital as long as the interest rate is not less than minus the depreciation rate, as capital depreciates without producing any output when entrepreneurs are not matched. As a result, unmatched entrepreneurs cannot borrow and they save using the risk-free bonds.

[10] Capital is predetermined, as in standard business cycle models. Note that this implies the investment decision of entrepreneurs is risky, as they choose the capital stock before observing realizations of idiosyncratic productivity shocks, as in Angeletos (2007) and David, Schmid, and Zeke (2022).

Matched entrepreneurs

At the core of our model there is the decision problem of matched entrepreneurs, whose solution characterizes the optimal wage contract. We denote by $J(m, W, z, S)$ their value function after matching and separation, according to equation (2). This depends on net worth m, idiosyncratic productivity z, utility promised to the worker W and the aggregate state of the economy S.

These entrepreneurs choose how much to consume this period, how much to borrow or save b' –where $b' > 0$ means they borrow–, and the capital stock that will be productive next period k'. They also choose how to fulfill their promise to the worker, meaning they decide how to deliver the utility W with state contingent wages and continuation values $w'(z', \xi'), W'(z', \xi')$.

The budget constraint of matched entrepreneurs at the end of period implies that the sum of consumption and physical capital has to be equal to the sum of net worth m and net borrowing qb'. The law of motion of net worth is given by the sum of output and the undepreciated capital stock minus wages paid to the worker and the repayment of outstanding debt. Borrowing is limited by a collateral constraint, so that firms can borrow up to a share ξ of their future capital stock. The promise keeping constraint makes sure the expected utility of the worker is at least equal to the promised utility W.

The value of being matched with a worker at the end of period is equal to the flow utility of consumption plus the expected continuation values of the entrepreneur. With probability $(1 - \phi)$ the match will survive until the end of next period, while with probability ϕ the match will separate and the entrepreneur will get the continuation value V of being vacant at the end of next period. We define the value V in (4).

$$\begin{aligned} J(m, W, z, S) = \max_{\substack{c^e, b', k', m'(z', \xi'),\\ w'(z', \xi'), W'(z', \xi')}} \Bigg\{ & v(c^e) + \beta(1-\phi) \underbrace{\mathbb{E}\left[J(m'(z', \xi'), W'(z', \xi'), z', S') | z, S\right]}_{\text{not separate}} \\ & + \beta\phi \underbrace{\mathbb{E}\left[V(m'(z', \xi'), z', S') | z, S\right]}_{\text{separate}} \Bigg\} \end{aligned} \tag{2}$$

$$\begin{aligned} &(\text{Budget constraint} : \lambda^e) && c^e + k' \leq m + qb' \\ &(\text{Net worth} : \eta(z', \xi')) && m'(z', \xi') \leq z' f(k') + (1-\delta)k' - w'(z', \xi') - b' \\ &(\text{Collateral constraint} : \mu) && b' \leq \xi k' \\ &(\text{Promise keeping} : \gamma) && W \leq \mathbb{E}\Big[u(w'(z', \xi')) + \beta(1-\phi)W'(z', \xi') + \beta\phi\mathcal{U}(S'') | z, S\Big] \end{aligned}$$

Unmatched entrepreneurs

Unmatched entrepreneurs face two decision problems: before matching and separation they can choose to open a vacancy to become matched by the end of the period, and after matching and separation they face a consumption/savings problem.

First, there is a discrete choice problem between posting a vacancy or not. Entrepreneurs who decide to open a vacancy have to choose a sub-market (θ, W) where to open it. With probability $\lambda_f(\theta)$ the entrepreneur is matched to a worker, where $J(m, W, z, S)$ denotes the value of a matched entrepreneur with utility W promised to the worker. With probability $1-\lambda_f(\theta)$ the entrepreneur remains vacant, where $V(m, z, S)$ denotes the value of a unmatched entrepreneur in the afternoon, after matching and separation. The entrepreneur opens a vacancy only if the expected continuation value from doing so is greater than the value $V(m, z, S)$ of being vacant at the end of period.

$$\widehat{V}(m, z, S) = \max \left(\max_{(\theta, W)} \left\{ [\lambda_f(\theta) J(m, W, z, S) + (1 - \lambda_f(\theta)) V(m, z, S)] \right\}, V(m, z, S) \right) \tag{3}$$

After matching and separation, unmatched entrepreneurs decide how much to consume and how much to save, according to (4). The value of being vacant at the end of period is equal to the flow utility of consumption plus the expected continuation value of being vacant next period, before matching and separation. At that stage, net worth will be equal to the returns on savings plus the flow value of home production $\bar{b}$.

$$V(m, z, S) = \max_{a', c^e, m'} \left\{ v\left(c^e\right) + \beta \mathbb{E} \left[\widehat{V}\left(m', z', S'\right) \mid z, S \right] \right\} \tag{4}$$

$$\text{(Budget constraint)}: \quad c^e + qa' \leq m$$

$$\text{(Net worth)}: \quad m' \leq a' + \bar{b}$$

Workers

Workers decide whether to search for a job, and if they search they choose a sub-market (θ, W) where to locate.

The value of a non employed worker before matching and separation, that we denote by $\mathcal{U}$, is defined in equation (5). First, they face a discrete choice problem between searching and not searching.

$$\mathcal{U}(S) = \max \Big(\underbrace{u(\bar{b}) + \beta \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid \mathcal{S}\right]}_{\text{if not search}}, \underbrace{u(\bar{b}(1-x)) + \beta \mathcal{W}(S)}_{\text{if search}} \Big) \tag{5}$$

The value of a non employed worker who does not search is equal to the flow utility of home production and the expected continuation value of being not employed next period. The value of a non employed worker who does search is given by the flow utility of home production, adjusted for the disutility cost of searching, and the expected continuation value of a worker who search, denoted by $\mathcal{W}(S)$ and defined in equation (6).

$$\mathcal{W}(S) = \max_{(\theta,W)} \{\lambda_w(\theta)W + [1-\lambda_w(\theta)]\, \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]\} \tag{6}$$

A non employed worker who searches in sub-market (θ, W) finds a job with probability $\lambda_w(\theta)$, and receives expected utility W next period conditional on finding a job. We say that a sub-market is active if there are at last some workers and some entrepreneurs searching in that sub-market. The problem of a worker who searches, as defined by equation (6), implies that workers search in a given sub-market $(\tilde{\theta}, \tilde{W})$ if and only if it is weakly better than searching in any other sub-market, that is:

$$\lambda_w(\tilde{\theta})\tilde{W} + \left[1-\lambda_w(\tilde{\theta})\right] \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right] \geq \max_{(\theta,W)} \{\lambda_w(\theta)W + [1-\lambda_w(\theta)]\, \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]\}$$

As all non-employed workers are homogeneous, the expected continuation value of a worker who searches $\mathcal{W}(S)$ must be equalized across all the active sub-markets.

2.3 Aggregation

We define aggregate output Y_t, capital K_t, employment N_t, and investment I_t as follows.[11]

$$Y_t = \int_0^1 y_{jt} \tag{7}$$

$$K_t = \int_0^1 k_{jt} \tag{8}$$

$$N_t = \int d\Lambda^m_{t-1}(m, W, z) \tag{9}$$

$$I_t = K_t - (1-\delta)K_{t-1} \tag{10}$$

We define aggregate debt B_t as the sum of gross debt of matched entrepreneurs:

$$B_t = \int \max\left(b(m, W, z, S_{t-1}), 0\right) d\Lambda^m_{t-1}(m, W, z)$$

Finally, we define aggregate productivity A_t such that:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

[11]Recall that $\Lambda^m_t(m, W, z)$ and $\Lambda^v_t(m, z)$ are the distributions of matched and unmatched entrepreneurs at the end of period t, after matching and separation occurred.

We can characterize aggregate productivity as a function of three terms, according to equation (11), where the expectation and covariance operators are taken with respect to the distribution of matched entrepreneurs Λ^m_{t-1} -normalized so that it adds up to one. The first term shows that A_t is proportional to the average productivity of active entrepreneurs $E[z_t]$. The second term, that is the ratio between $E[k^\alpha]$ and $E[k]^\alpha$, shows that aggregate productivity is lower when the cross-sectional dispersion in capital is higher. Finally, the covariance term implies that A_t is larger when more productive entrepreneurs produce a larger share of aggregate output.

$$A_t = E[z_t]\left[\frac{E[k^\alpha]}{E[k]^\alpha}\left(1+\frac{Cov(z,k^\alpha)}{E[z]E[k^\alpha]}\right)\right] \tag{11}$$

More precisely, let us denote by $\tilde{\Lambda}^m_{t-1}$ the normalized distribution of matched entrepreneurs. We have that:

$$A_t = \int\sum_{z'} z'\Pi(z'|z)d\tilde{\Lambda}^m_{t-1}(m,W,z)\left[\frac{\left(\int k(m,W,z)^\alpha d\tilde{\Lambda}^m_{t-1}(m,W,z)\right)}{\left(\int k(m,W,z)d\tilde{\Lambda}^m_{t-1}(m,W,z)\right)^\alpha}\left(1+\frac{Cov(z,k^\alpha)}{E[z]E[k^\alpha]}\right)\right] \tag{12}$$

2.4 Competitive equilibrium

Definition 1. *A recursive competitive equilibrium is defined as : i) a law of motion Γ for the aggregate state S, ii) entrepreneurs' policy functions and value functions, iii) workers' policy functions and value functions, iv) distributions of matched and unmatched entrepreneurs $\Lambda^m(m,W,z), \Lambda^v(m,z)$ v) price q and market tightness $\{\theta\}$ in active sub-markets, such that:*

- *non-employed workers solve their problem given Γ, $\{(\theta, W)\}$*
- *entrepreneurs solve their problem given ,$\mathcal{W}, q, \Gamma$*
- *law of motion Γ is consistent with the policy functions and the value functions*
- *the measure of workers who search is consistent with $\{\theta\}$*
- *q is such that the bond market clears*

$$\int b'(m,W,z,S)d\Lambda^m(m,W,z) = \int a'(m,z,S)d\Lambda^v(m,z)$$

In Appendix A we show that the definition of equilibrium implies the resource constraint from Walras' Law, and we define formally the law of motion Γ for the aggregate state.

Solving this model poses some challenges, as the decision problems of matched and unmatched entrepreneurs depend on $q, \mathcal{W}(S), \mathcal{U}(S)$, that are all endogenous objects, that depend on the exogenous aggregate state ξ as well as the endogenous state S. In Proposition 1, we show that the values $\mathcal{W}, \mathcal{U}$ do not depend on the aggregate state of the economy S.[12] This result substantially simplifies the analysis we conduct in Section 5, as the problem of entrepreneurs depends on S only through (ξ, q).

Proposition 1. *If the measure of workers M is large, in equilibrium these properties hold: (i) a positive measure of workers does not search (ii) non employed workers are indifferent between searching and not searching (iii) the values $\mathcal{W}(S)$ and $\mathcal{U}(S)$ do not depend on the aggregate state S, (iv) the value functions of entrepreneurs depend on S only through (ξ, q).*

Proof: See Appendix A

2.5 Discussion

Before moving on, let us discuss some of the assumptions we made.

First, we assume that financial friction takes the form of a collateral constraint, consistent with a large literature built on Kiyotaki and Moore (1997). This model assumption suits particularly well a country such as Italy, where firms rely heavily on bank financing and where bank credit is largely collateralized (Garrido, Kopp, and Weber, 2016; Affinito, Sabatini, and Stacchini, 2021). We could alternatively model financial frictions as a working capital constraint, in the spirit of Jermann and Quadrini (2012) or Bocola and Lorenzoni (2023), where firms raise funds with intra-period loans to purchase inputs. This alternative modeling assumption would likely amplify the effects of wage backloading on investment. Indeed, relatively lower wages would not only free up resources for investment by increasing net worth but also enhance borrowing capacity, as entrepreneurs would need to borrow less to pay the wage bill.

Second, we assume that entrepreneurs -firms' owners- have a concave utility function and cannot issue equity. Although this assumption is fairly common in models of financial friction (Moll, 2014; Kiyotaki and Moore, 2019), our mechanism would extend to an economy with risk-neutral entrepreneurs (Khan and Thomas, 2013) who can issue equity subject equity issuance costs, as in Jermann and Quadrini (2012). Wage contracts would still solve a risk-sharing problem, where wages co-move with the marginal value of a dollar for the entrepreneur.[13]

Third, we assume that workers and firms can commit to a wage contract. Firms' commitment is a common assumption in the literature studying dynamic wage contracts,

[12] For this step it is crucial to show that the values $\mathcal{W}, \mathcal{U}$ do not depend on any endogenous aggregate state. Indeed, if these values were functions of exogenous aggregate states only, they would be easy to forecast and they would not pose any computational challenge to solving the model.

[13] In this class firms don't distribute dividends and don't consume, as long as there is a positive probability of facing a binding borrowing constraint at any time in the future.

as in Harris and Holmstrom (1982) and Balke and Lamadon (2022), and it is often motivated by firms' reputational concerns. Moreover, this assumption suits well a country like Italy where firing workers is costly.[14] Here we assume that workers can also commit to a contract, as in Boldrin and Horvath (1995), Schaal (2017). Intuitively, workers' limited commitment would not impair firms' ability to temporarily backload wages when access to credit is limited. Indeed, when wages are backloaded, workers are promised higher wages in the future, and therefore they have incentives to stay in the match even under limited commitment. We propose an extension of the model subject to limited commitment by workers in Appendix D.1.

Fourth, we assume that each entrepreneur can hire only one worker. Despite this assumption is common to several search models of the labor market, it is not without loss of generality in a model with investment and search frictions. For instance, firms' ability to adjust wages of incumbent workers might affect the decision of hiring new employees within the same multi-worker firms. While this would be an interesting extension, the implied contracting problem would be intractable with an almost infinite dimensional state variable, since one would have to keep track of the promised utility offered to each worker.[15]

Fifth, we assume that the workers are hand-to-mouth. This assumption is common to several search models of the labor market and models of dynamic wage contracts and is consistent with the view that firms have better access to financial markets than workers. Note that if workers had unrestricted access to financial markets, then entrepreneurs would implicitly borrow from their employees to completely offset the effects of financial frictions. It would be interesting to consider an intermediate case, where workers could save and borrow subject to some borrowing constraint (Souchier, 2024). In this setting, firms could implicitly borrow from their employees even more than they do under our stark assumption of hand-to-mouth workers, potentially giving firms more room to temporarily backload wages after a credit tightening.

3 Model mechanism

This section characterizes the properties of the optimal wage contract and illustrates the main mechanism of the model.

[14] Regulations in the Italian labor market make it particularly costly to layoff workers for firms with more than 15 employees, as those observed in the data that we discuss in Section 4. Layoffs and discharges are much rarer in Italy than in countries such as the United States. The average annual rate of layoffs and discharges in Italy is approximately 2%, while in the United States it is greater than 10% and separations are an important driver of earnings fluctuations (Meeuwis et al., 2023).

[15] Workers hired in different years will have different promised utilities, so one would have at least one state variable for each cohort of incumbent workers. The problem can be made tractable with some specific assumptions, as in Michelacci and Quadrini (2009), which consider an environment without persistent idiosyncratic shocks and aggregate shocks, but the general case is not tractable.

In Section 3.1 we consider a special case of our model that is analytically tractable. We show that the optimal wage contract takes a simple form: wages are a constant share of entrepreneurs' net worth over the length of an employment relationship. This tractability allows us to explicitly characterize how wages adjust over time and in response to shocks as a function of the leverage of entrepreneurs, that is, b/k. We focus on leverage, as in our model, firms with higher leverage are more likely to be financially constrained. We show that entrepreneurs with high leverage adjust wages more in response to idiosyncratic shocks and increase wages more over time.

In Section 3.2 we turn to the general case. We use optimality conditions to illustrate the determinants of the investment, consumption, and savings decision. Then, we explain the main trade-off in wage setting, highlighting similarities with the special case. Both cases share the same underlying logic, which accounts for the heterogeneity in wage dynamics across firms. We illustrate that in the general case entrepreneurs that are more financially constrained *i)* temporarily backload wage payments (that is, pay a steeper wage-tenure profile), *ii)* temporarily backload wages even more during recessions, and *iii)* adjust wages more in response to idiosyncratic shocks.

Finally, Section 3.3 highlights the link between the structure of wage contracts, investment, and job creation, by considering the problem of unmatched entrepreneurs.

3.1 Special case: analytical results

In this section we consider a special case of the problem of matched entrepreneurs that is analytically tractable. This special case allows us to solve analytically for the policy functions of consumption and wages and to characterize how wage dynamics vary with entrepreneurs' financial conditions. We consider an economy where both entrepreneurs and workers have log utility, the production technology is linear in capital, and there is no separation.

$$v(c_t^e) = \log(c_t^e), \qquad u(w_t) = \log(w_t), \qquad f(k_t) = k_t, \qquad \phi = 0$$

Combining the optimality conditions for state-contingent wages and workers' continuation values we obtain a risk-sharing condition, according to equation (13). Indeed, because both entrepreneurs and workers can commit to future wages, a wage contract offers a full set of state-contingent claims within the employment relationship that implies perfect risk-sharing. Since we assume that entrepreneurs and workers have the same preferences, this risk-sharing problem implies that wages move one-for-one with the entrepreneur's consumption.

$$\frac{c^e\left(z^{t+s},\xi^{t+s}\right)}{w\left(z^{t+s},\xi^{t+s}\right)} = \frac{c^e\left(z^{t+k},\xi^{t+k}\right)}{w\left(z^{t+k},\xi^{t+k}\right)} \quad \forall z^{t+s}, z^{t+k}, \xi^{t+s}, \xi^{t+k} \tag{13}$$

The next proposition characterizes the policy functions for consumption and wages.

Proposition 2. *Entrepreneur's consumption and worker's wage are linear in net worth:*

$$c_t = (1-x)m_t, \qquad w_t = \gamma(1-x)m_t$$
$$\textit{with} \quad 1-x = \frac{1-\beta}{1+\beta\gamma}$$

where γ is the multiplier on the promise keeping constraint, that is constant over time.

Proof. See Appendix A.

Proposition 2 shows that wages are a constant share of entrepreneurs' net worth over the length of an employment relationship. This constant share depends on the discount factor β and the multiplier on the promise-keeping constraint γ. If entrepreneurs initially promised a higher utility W to the worker, the multiplier γ will be higher. Intuitively, this result implies that entrepreneurs pay relatively lower wages over the course of a match then their net worth is low, and that wages are expected to grow and adjust over time with net worth.

An immediate corollary of this result is that wages increase when idiosyncratic productivity increases, as that leads to an increase in future net worth of the entrepreneur.

Corollary 1. *State contingent wages $w(z_{t+s}|z^{t+s-1})$ are increasing in productivity z_{t+s}.*

The result in Proposition 2 holds regardless of whether the collateral constraint binds. We now turn to studying how wage dynamics depends on financial frictions. To this end, we define leverage as the ratio between entrepreneurs' debt and capital, that is b/k. Due to collateral constraints, leverage must be below the collateral value of capital ξ, and entrepreneurs with higher leverage are closer to facing a binding constraint. The next proposition characterizes how wages change both in response to idiosyncratic shocks and over time depending on the leverage of the entrepreneur.

Proposition 3. *For each time t, the pass-through of idiosyncratic productivity shocks to wages, defined in (14), and the expected growth rate of wages, defined in (15), are increasing in entrepreneurs' leverage b_{t+1}/k_{t+1}.*

$$\frac{\partial[\log(w_{t+1}) - \log(w_t)]}{\partial[\log(z_{t+1}) - \log(z_t)]} \tag{14}$$

$$\mathbb{E}[\log(w_{t+1}) - \log(w_t)] \tag{15}$$

Proof. See Appendix A.

Proposition 3 implies that highly levered entrepreneurs adjust wages more in response to idiosyncratic shocks, as for the same change in productivity they adjust wages more. Intuitively, this result is implied by the law of motion of net worth and Proposition 2. Because entrepreneurs borrow using uncontingent debt, a higher leverage makes their net worth in $t+1$ more sensitive to fluctuations in future productivity z_{t+1}. As their net worth fluctuates more, so must the wage according to Proposition 2.

Proposition 3 also implies that the expected growth rate of wages is higher for entrepreneurs with higher leverage. Intuitively, because leverage is endogenous, highly levered entrepreneurs choose to borrow more and invest more in capital relative to their net worth. This means that highly levered entrepreneurs must expect higher returns from their investment, and thus they promise higher returns to their workers in the form of future wage growth.

Note that the results of Proposition 3 apply to a cross-section of firm at any given point in time. This means that in normal times wages grow faster at more levered entrepreneurs, but also that during recessions wages grow faster at more levered entrepreneurs. We will return to this point in the general case.

3.2 General case

We now turn to the problem of a matched entrepreneur in the general case. First, we discuss the optimal choice of investment, debt, and consumption using the optimality conditions. Then, we illustrate how wage dynamics depends on entrepreneurs' financial conditions, emphasizing how the general case shares the same underlying logic underneath the analytical result from Section 3.1.

3.2.1 Investment, debt, and consumption

The recursive problem of matched entrepreneurs defined in (2) implies fairly common optimality conditions for debt and capital:

$$v'(c^e) = \frac{\mu}{q} + \frac{1}{q}\mathbb{E}\left[\eta\left(z', \xi'\right)\ |z, \xi\right] \tag{16}$$

$$v'(c^e) = \mu\xi + \mathbb{E}\left[\eta\left(z', \xi'\right)\left[z' f'(k') + 1 - \delta\right]\ |z, \xi\right] \tag{17}$$

where μ denotes the multiplier on the collateral constraint, $\eta(z', \xi')$ denotes the multiplier on the law of motion of net worth that is the marginal value of a dollar for the entrepreneur in state (z', ξ').

In the general case we have $f(k) = k^{\alpha}$. Compared to the special case, the marginal product of capital is decreasing in k. This implies there is always a positive and fined first-best level of capital, according to equation (18), that we denote by k^{FB}.[16]

$$\frac{1}{q} - (1-\delta) = f'(k^{FB})\mathbb{E}[z'|z] \tag{18}$$

Because of financial frictions, capital is always below the first best level, that is $k' < k^{FB}$. Indeed, distortions appear in the optimality condition for capital, that can be rearranged to obtain (19) by combining (16) and (17). The presence of borrowing constraint on the left hand-side of (19) reduces entrepreneurs' capital whenever the constraint is binding, that is $\mu > 0$. The presence of uninsurable idiosyncratic risk lowers the returns to capital on the right hand-side of (19), which are equal to the product of the risk-neutral marginal product of capital and a risk adjustment term. This is because, as in the seminal work of Guiso and Parigi (1999), investing in capital is risky as entrepreneurs' consumption is correlated with their idiosyncratic productivity. This risk premium term reduces returns to capital because the marginal value of a dollar for the entrepreneur, $\eta(z',\xi')$, is negatively correlated with future productivity z'.

$$\underbrace{\frac{\mu}{\mathbb{E}\left[\eta\left(z',\xi'\right)|z,\xi\right]}\left(\frac{1}{q}-\xi\right)}_{\text{borrowing constraint}} + \frac{1}{q} - [1-\delta] = f'\left(k'\right)\mathbb{E}\left[z'|z\right]\underbrace{\left[1-\left|\frac{\mathrm{Cov}\left(\mathbb{E}\left[\eta\left(z',\xi'\right)|z',\xi\right],z'|z\right)}{\mathbb{E}\left[\eta\left(z',\xi'\right)|z,S\right]\mathbb{E}\left[z'|z\right]}\right|\right]}_{\text{idiosyncratic risk}} \tag{19}$$

Entrepreneurs with low values of net worth face a binding collateral constraint. Indeed, they are less able to self-finance investment in capital and *ceteris paribus* they need to borrow more. For higher level of net worth the collateral constraint does not bind, and these entrepreneurs are close to the first-best level of capital. Moreover, as the collateral constraint is eased, the leverage of entrepreneurs decreases, and entrepreneurs with higher net worth build a sufficiently high buffer stock of savings to insure against idiosyncratic shocks. Therefore, the covariance term on the right-hand side of (19) decreases with net worth, making capital even closer to k^{FB}.

3.2.2 Risk sharing

Equation (20) is the optimality conditions for the state-contingent wages $w'(z',\xi')$. On the left-hand side of (20), the marginal cost of increasing $w'(z',\xi')$ is equal to the marginal value of a dollar for the entrepreneur in that state of the world. Intuitively, the marginal value of a dollar for the entrepreneur is high when net worth is low, that is, when the entrepreneurs' level of capital is likely below the first best level. On the right-hand side,

[16]The first-best level of capital is the optimal choice of entrepreneurs in an economy with complete markets. This means there are no borrowing constraints, entrepreneurs can issue state-contingent claims to perfectly insure against idiosyncratic shocks. Note that in the first-best case there are no aggregate financial shocks, as borrowing is unconstrained.

the benefit of increasing wages is large in states where workers' marginal utility is low (i.e., the worker value insurance against income risk) and proportional to the multiplier γ on the promise keeping constraint.

$$\eta(z', \xi') = \gamma u'(w'(z', \xi')) \tag{20}$$

$$\gamma = \gamma(z', \xi') \tag{21}$$

Crucially, (20) implies a trade-off between smoothing earnings of risk-averse workers –from the right hand-side– and smoothing the marginal value of a dollar in the firm to relax the effects of financial frictions –from the left hand-side–. If the collateral constraint binds, smoothing the marginal value of a dollar in the firm means investing more in capital in the short term to ease the constraint. If the collateral constraint does not bind, smoothing output means building a buffer stock of savings to avoid binding constraints in the future.

As in the special case, the optimal wage contracts between entrepreneurs and workers can be described as the solution to a risk-sharing problem. The ratio between the marginal utility of the worker and the marginal value of a dollar for the entrepreneur is equal to γ, which does not depend on future states (z', ξ'). Moreover, the optimality conditions for the state-contingent continuation values $W'(z', \xi')$ imply that γ has to be constant over time, according to equation (21). Therefore, as long as the entrepreneur and the worker do not separate, the optimal wage contract implies perfect risk-sharing with respect to idiosyncratic productivity shocks and aggregate financial shocks.[17] Since the consumption of entrepreneurs depends on their net worth, wages also depend on net worth because of optimal risk sharing, in a way that recalls the special case.

Unlike the special case, the degree of risk sharing implied by wage contracts depends on the relative risk aversion coefficients of entrepreneurs and workers, σ_E and σ_W. Under the assumption of CRRA preferences, σ_W captures both the relative risk-aversion coefficient and the inverse elasticity of inter-temporal substitution. In Appendix D.2 we characterize the solution of the risk-sharing problem under the assumption that workers have Epstein-Zin preferences (Epstein and Zin, 1989), and we highlight the role played by risk-aversion and inter-temporal substitution. Interestingly, the ability of firms to adjust wages over time depends both on the elasticity of inter-temporal substitution and the degree of risk-aversion of workers. Indeed, future earnings are uncertain due to the risk of separation, and wage dynamics depend on workers' preference towards persistence risk, that depends both on risk aversion and inter-temporal substitution.

[17] Since wages cannot be paid after separation, entrepreneurs and workers are still exposed to the idiosyncratic risk of separation.

3.2.3 Wage dynamics and firms' financial conditions

In the special case presented in Section 3.1 wage dynamics depend on leverage: more constrained entrepreneurs (i.e., higher leverage) adjust wages more in response to idiosyncratic shocks and offer higher future wage growth. We know illustrate the mechanism in more detail for the general case, where entrepreneurs with low net worth are *ceteris paribus* more financially constrained, as they are less able to self-finance investments.[18]

To illustrate the mechanism, consider two entrepreneurs that start at time $t = 0$ with different net worth m_0, but with the same utility promised to the worker and the same productivity (W_0, z_0) .Figure 1 plots whether the constraint binds, wages, and capital as a function of time for these two entrepreneurs.

Panel (a) considers a case where no shocks are realized. In this case, the entrepreneur that starts with high net worth is unconstrained, and both capital and wages are almost flat over time. On the other hand, the constraint binds for the entrepreneur that starts with a lower net worth, and this entrepreneur offers a temporarily steeper wage tenure profile. This means wages are lower initially, but they increase more over time as the entrepreneur accumulates net worth and approaches the first best level of capital. Intuitively, this entrepreneur temporarily backloads wages by paying workers more in the future than today.[19] This property of wage contracts aligns with the findings of Michelacci and Quadrini (2009), that studies optimal wage contract when firms are subject to financial constraints in a deterministic environment.

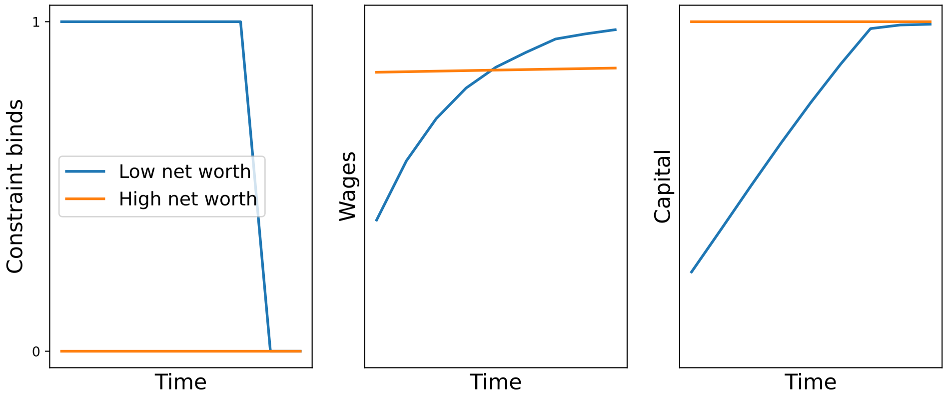
Consider now an aggregate financial shock. Panel (b) plots whether the constraint binds, wages, and capital as a function of time, when there is a one-time financial shock at $t = 0$ (dashed line). The entrepreneur with high net worth is only mildly affected by the shock, that leads to small changes in wages and capital. The entrepreneur with low net worth is greatly affected by the shock, leading to a substantial contraction in current and future investment. In response to the shock, the entrepreneur with low net worth makes the wage-tenure profile of the worker steeper, thus backloading wages more after an aggregate financial shock.

The dashed lines in Panel (c) plot whether the constraint binds, wages and capital as a function of time when there is a positive idiosyncratic productivity shock at time $t = 0$.[20] The mechanism is the same underneath Corollary 1 and Proposition 3. An increase in productivity leads to higher net worth and higher wages, because of optimal risk-sharing. The entrepreneur with low net worth is more sensitive to changes in idiosyncratic

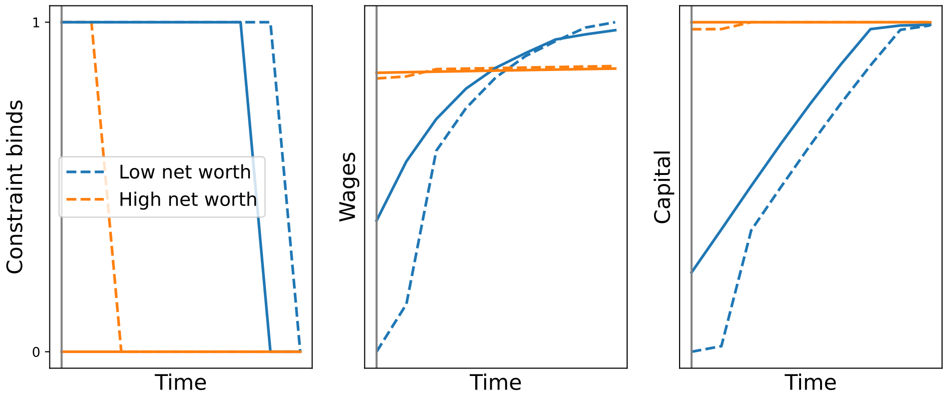
[18] This means that in the entrepreneurs with low net worth have higher leverage, so that by emphasizing the link between wage dynamics and net worth we also illustrate how the findings from the special case extend to the general case.

[19] Since in this example both workers receive the same expected utility W_0, but the timing of wage payments differ for these workers, we say that the entrepreneur with low net worth backloads wages.

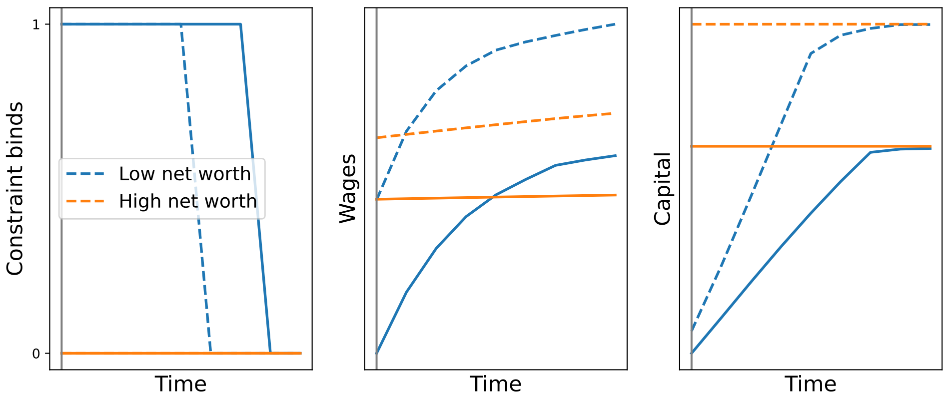
[20] We consider a permanent increase in productivity that occurs at period $t = 0$, so that productivity increases and stays higher in future periods.



(a) No shocks



(b) Aggregate financial shock: $\xi \downarrow$



(c) Idiosyncratic productivity shock: $z \uparrow$

Figure 1: Wage-tenure profile, capital and net worth dynamics for entrepreneurs with same (W, z) but different net worth m in the first period. Wages, capital and net worth are plotted an entrepreneur with low net worth (blue line) and high net worth (orange line).

productivity, as higher leverage makes his returns more correlated with fluctuations in output. Consequently, entrepreneurs with low net worth adjust wages more in response to idiosyncratic shocks.

3.3 Job creation, investment, and wages

The dynamic structure of wage contracts, namely the fact that entrepreneurs can adjust wages both over time and in response to shocks, affect both investment and job creation. First, we discuss the implications for investment, relating to the problem of matched entrepreneurs. Then, we characterize the optimal job creation decision of unmatched entrepreneurs and explain how it depends on the dynamic structure of wage contracts.

3.3.1 Investment and wages

The ability of firms to adjust wages both over time and in response to shocks enhances investment. First, the ability to backload wages over time allows entrepreneurs that are currently financially constrained to pay lower wages in the short term so to have more resources to invest in capital. Additionally, as illustrated in (20), wage contracts determine the degree of risk sharing between the entrepreneur and the worker. When an entrepreneur shares some idiosyncratic risk with the worker, all else equal, it decreases his exposure to idiosyncratic risk, thus making investment less risky and more attractive.

To understand these effects, consider an alternative economy in which wages are determined on a spot labor market. Spot market wages would not be contingent on firms' idiosyncratic productivity simply because of the law of one price. As a result, entrepreneurs would bear the entire investment risk, making capital less attractive. Similarly, if wages were determined on a spot labor market, entrepreneurs would not adjust the wage-tenure profile of workers according to their financial conditions.

Importantly, even a small change in the current wage can have large effects on the entire investment dynamics. Consider an entrepreneur with a binding constraint that offers a lower wage at time t. The current investment increases mechanically as more resources become available. If the collateral constraint also binds in future periods, future investment also increases. Indeed, higher investment in t increases net worth in $t+1$, which further enhances investment in $t+1$, thus increasing net worth in $t+2$, and so on.[21] As highlighted in Khan and Thomas (2013) and Moll (2014), the effects of financial frictions are persistent both in recession and in the cross-section, suggesting that a small change in wages can have persistent effects on investment. This dynamic effect of current wages on future investment has implications for job creation that we illustrate in the next section.

[21] Note that if an entrepreneur becomes unconstrained in $t+1$, higher earnings in $t+1$ will have little effects on investment in $t+1$ as the entrepreneur would be able to self-finance himself.

3.3.2 Job creation

Job creation is determined by the solution of the problem of unmatched entrepreneurs, according to (66), who decide whether to open a vacancy and a sub-market in which to post. The latter decision implies the job creation equation (22) that mimics a standard surplus sharing rule common to a large class of search models.[22] On the left-hand side of (22) there is the ratio between the surplus of the entrepreneur, that is $J - V$, and the surplus of the worker, that is $W - \mathcal{U}$. On the right-hand side there is the product between a standard term in search models that reminds of the Hosios' condition[23] and the multiplier on the promise keeping constraint γ, that intuitively captures how costly it is for the entrepreneur to deliver the promised utility W.

In the standard competitive search model the path of wages is not uniquely pinned down, as standard assumptions on risk-sharing and complete financial markets make it irrelevant in terms of the equilibrium allocation. In this model, because of incomplete markets, the specific path of wages is allocative for job creation and thus affects the equilibrium allocation. We emphasize two ways in which the dynamic structure of wage contracts affects job creation.

$$\frac{J(m, W, z, S) - V(m, z, S)}{W - \mathcal{U}} = \underbrace{\left(-\frac{\partial J(m, W, z, S)}{\partial W}\right)}_{\gamma(m, W, z, S)} \left(\frac{1-\eta}{\eta}\right) \tag{22}$$

First, the structure of wage contracts affects job creation through the surplus $(J - V)$. For example, when an entrepreneur with low net worth backloads wages to enhance investment, he will make the worker more productive over the length of a match increasing the surplus $(J - V)$. In the previous section, we emphasized that even a small change in current wage can have large effects on the dynamics of investment when credit constraints bind for several periods. Crucially, job creation that depends on the present discounted value of a match and thus internalizes these dynamics effects of wages on investment.

Second, the entrepreneur optimally sets wages in order to minimize the present discounted value of all future wage payments made to the worker. Intuitively, the ability to make state-dependent wage adjustments in the future decreases the cost of hiring a worker in present discounted value terms today, thus fostering job creation. The following lemma formalizes this result.

[22] Appendix A derives equation (22) from the problem of unmatched entrepreneurs using Proposition 1.

[23] In the efficient allocation of a standard search model, the ratio between firms' surplus and workers' surplus is equal to the ratio $(1 - \eta)/\eta$. See Hosios (1990).

Lemma 1. *The optimal wage contract that solves* (2) *is also a solution to:*

$$\min_{\{w_{s+1}(z^{s+1},\xi^{s+1})\}_{s=t}^{\infty}} \mathbb{E}\left[\sum_{s=t}^{\infty} w_{s+1}(z^{s+1},\xi^{s+1}) \times [\beta(1-\phi)]^{(s-t)} \frac{\eta_{s+1}(z^{s+1},\xi^{s+1})}{v'(c_t^e)}\right] \tag{23}$$

$$\textit{subject to:} \quad W \leq \mathbb{E}\left[\sum_{s=t}^{\infty} [\beta(1-\phi)]^{(s-t)} u(w_{s+1}(z^{s+1},\xi^{s+1})) \; + \; [\beta(1-\phi)]^{(s-t)}\beta\phi\mathcal{U}\right]$$

Proof: See Appendix A

4 Empirical evidence

We use matched employer-employee data from Italy to provide empirical evidence supporting predictions on wage dynamics implied by the model.

In Section 3.1, we showed that financially constrained firms adjust wages more in response to idiosyncratic shocks and temporarily backload wages of their workers. The extent to which constrained firms temporarily backload wages of their workers is particularly pronounced after an aggregate financial shock. For newly created matches, this means constrained firms offer a temporarily steeper wage-tenure profile, and they make it even more steeper in response to a credit tightening. For incumbent workers, this implies that constrained firms reduce the growth rate of wages of these workers in response to a credit tightening.

In the empirical analysis, we validate these key predictions of the model: financially constrained firms adjust wages more in response to idiosyncratic shocks, they offer temporarily steeper wage-tenure profiles during recessions, and wage growth of their incumbent workers slow down more during recessions. Testing these predictions is crucial for two reasons. First, the differential wage response to idiosyncratic shocks provides evidence for the *risk-sharing* mechanism underneath wage-setting. In practice, we estimate that financially constrained firms display higher pass-through of value added per worker to wages than unconstrained firms. We focus on pass-through coefficients as a measure of risk-sharing between firms and workers, building on the work of (Guiso, Pistaferri, and Schivardi, 2005; Guiso and Pistaferri, 2020). Second, the temporarily steeper wage-tenure profiles offered by constrained firms to workers hired during recessions highlight the *dynamic* nature of wage adjustments over the length of employment relationships, while emphasizing how firms optimally adjust wages to ease the effects of an aggregate tightening.

Since identifying financially constrained firms is notoriously difficult, we show that our results are robust to using several methods to classify firms as more or less financially constrained. In the baseline specifications we consider firms with higher leverage to be more financially constrained. This is consistent with both the model and several studies that use leverage as a proxy for the strength of financial frictions across firms (Gopinath et al., 2017; Ottonello and Winberry, 2020). Moreover, leverage is a particularly suitable proxy in countries like Italy, where firms rely heavily on bank financing and where bank credit is largely collateralized (Garrido, Kopp, and Weber, 2016; Affinito, Sabatini, and Stacchini, 2021). Furthermore, evidence shows that firms with higher leverage were more significantly impacted by the Great Recession and the EU Sovereign Debt crisis, as they rely more on external financing and were therefore more affected by a credit tightening (Arellano, Bai, and Bocola, 2017; Kalemli-Özcan, Laeven, and Moreno, 2022). Leverage also correlates with other commonly used proxies for financial constraints, such as credit scores and distance to default (Rodano, Serrano-Velarde, and Tarantino, 2018; Ottonello and Winberry, 2020).

We find similar results using several alternative methods to classify firms as financially constrained. In a first set of robustness exercises, we classify firms as financially constrained using other common indicators constructed from balance sheet data. More precisely, we identify financially constrained firms depending on their debt-to-output ratio and interest coverage ratio. This approach is motivated by evidence that these ratios are central to commonly used loan covenants, restricting borrowing for firms with low earnings relative to their debt or interest payments (Greenwald, 2019; Drechsel, 2023). In a second set of robustness exercises, we classify firms as financially constrained based on heterogeneous fluctuations in local credit supply during the Great Recession (Greenstone, Mas, and Nguyen, 2020; Barone, de Blasio, and Mocetti, 2018). More precisely, a shift-share style research design is implemented to isolate local lending shocks using variation in preexisting bank market shares and bank supply shifts. In this way, we identify as more financially constrained those firms operating in local credit markets that experienced a larger credit supply shock in 2008.

The section is organized as follows. First, we describe the data and the institutional background. Then, Section 4.4 presents the empirical strategy and the main results on how firms temporarily backload wages of workers hired in the Great Recession, Section 4.5 documents evidence on the wage growth of incumbent workers during the Great Recession, and Section 4.6 presents results on heterogeneous pass-through coefficients.

4.1 Matched employer-employee data

The analysis relies on matched employer-employee data sourced from official social security records maintained by the Italian Social Security Institute (Istituto Nazionale Previdenza Sociale, INPS). This comprehensive dataset, spanning from 2005 to 2019, includes the entire employment history of all private-sector employees who were at any time employed by firms participating in the Survey of Industrial and Service Firms (INVIND survey) conducted by the Bank of Italy. The relevant population for the INVIND survey includes firms in manufacturing and services, with at least 20 employees. Known as the INPS-INVIND dataset, it integrates employer and employee data, covering approximately 10 million workers and 10 thousand firms over the 2005-2019 period.[24]

From INPS records, for each job spell in every year, we observe worker and firm identifiers, along with gross earnings, the number of weeks worked in full time equivalent units, part-time status and a coarse occupational code (apprentice, blue collar, high-skilled blue collar, white collar, middle manager or manager). For each worker we also observe a series of basic demographic characteristics such as gender and year of birth. We complement the INPS-INVIND data by matching it to balance sheet and income statement information from CERVED.[25] For each firm in the sample we retrieve firms' total assets, value added and various measures of firm debt, including debt with banks, suppliers, or other intermediaries. These variables are particularly important as they allow us to construct different measures of leverage. We describe in more details the data, the cleaning procedure, and the sample construction in Appendix B.

In some robustness exercises, we use the local credit supply index constructed in Barone, de Blasio, and Mocetti (2018), who use granular data on bank lending from the Bank of Italy Supervisory Report database, and data from INVIND survey questions, such as the total share of overtime hours at the firm level.

4.2 Institutional background

The relevant tiers for wage formation in the Italian labor market are at the industry and company levels (Guiso, Pistaferri, and Schivardi, 2005). The first pillar consists of sectoral bargaining agreements at the industry level that establish "minimum wages" (contractual minimums, or *minimi tabellari*) for different job ladder levels. The second pillar consists of company-specific components of the compensation package, allowing the firm to unilaterally determine certain elements or negotiate them in a company contract with unions. The most significant portion is the company-level wage increment (*super-minimum*), which permanently raises the contractual minimum wage in nominal terms and includes both firm- and worker-level components. Additional forms of firm-specific compensation include transitory production bonuses (*premi di produzione*) and a variable pay component (*retribuzione variabile*).

A substantial share of the workers' compensation package is determined at the firm level. Using data from 1975 to 2000, Card, Devicienti, and Maida (2014) shows that nearly

[24]The INPS-INVIND data have been used in a number of recent studies, including (Macis and Schivardi, 2016; Daruich, Di Addario, and Saggio, 2023; Di Addario et al., 2023) among others.

[25]CERVED is a leading Italian data provider, offering detailed balance sheet data and comprehensive business information.

all employees earn some premium above the contractual minimum, with the median premium being 24%. Guiso, Pistaferri, and Schivardi (2005) reports that in 1994 the average wage component due to firm-specific pay policies was around 23%. The latter grew to around 30% in 2009 according to the same data source (Daruich, Di Addario, and Saggio, 2023). Since then, the Italian labor market has witnessed a gradual erosion of centralized bargaining agreements (D'Amuri and Giorgiantonio, 2015). This process culminated in the Inter-sectoral Agreement of June 2011 which broadened the scope of decentralized wage-bargaining and defined the procedures for its activation. Meanwhile, Law 148/2011 introduced the possibility of signing firm and local-level agreements in derogation of the law and of the national collective agreements.

4.3 Summary statistics

Table 1 presents summary statistics for the distribution of wage growth among incumbent workers. For this and all empirical exercises, we calculate annual wage growth for full-time, year-round workers employed at the same firm in two consecutive years, ensuring stable working hours.[26] Importantly, our measure of wages includes the base wage (*superminimum*), but also production bonuses and a variable pay component, all of which contribute to both firms' labor costs and workers' earnings, represented in the model by wage w. We evaluate the average wage growth at the firm level to remove volatility in wages due to worker-specific idiosyncratic shocks.

Table 1 reports several moments of the distribution of wage growth before the Great Recession, in 2007, and during the Great Recession, in 2009. We report statistics for both nominal wage growth, in Panel A, and for real wage growth, in Panel B. First, the average growth rate of wages was substantially lower during the Great Recession in both nominal and real terms. Moreover, less than 10 percent of workers experienced negative nominal wage growth in 2007, but this share increased to approximately 20 percent in 2009. These magnitudes are consistent with recent evidence from the United States documented by Grigsby, Hurst, and Yildirmaz (2021).[27] Furthermore, the skewness of the wage growth distribution decreased sharply during the Great Recession, consistent with previous evidence from Italy documented by Adamopoulou et al. (2016).

[26] In Section 4.4 we discuss how changes in overtime hours may influence the results, and in Appendix B.5 we show that our main results are not driven by fluctuations in overtime hours.

[27] Using administrative payroll data, Grigsby, Hurst, and Yildirmaz (2021) finds that approximately 17 percent of workers saw declines in average hourly earnings, once accounting for adjustments of the base wage and other components of workers' compensation. Moreover, they document that the share of workers experiencing a decrease in their base wage was much higher during the Great Recession.

Summary statistics: wage growth $\times 100$						
Panel A: Nominal wage growth						
	Mean	Std. deviation	p10	p50	p90	Skewness
2007	3.5	2.5	0.2	3.5	6.3	0.68
2009	0.9	4.7	-5.7	1.9	5.5	-1.47
Panel B: Real wage growth						
	Mean	Std. deviation	p10	p50	p90	Skewness
2007	1.7	2.5	-2.0	1.7	4.5	0.68
2009	0.2	4.7	-6.3	1.2	4.8	-1.47

Table 1: Average wage growth at the firm level for incumbent workers employed full-time and year-round in two consecutive years. Each firm is weighted by the number of its employees.

4.4 Wage backloading in the Great Recession

We now discuss the estimation of wage-tenure profiles for workers employed by firms with different levels of leverage, to provide empirical evidence that firms with high leverage offered steeper wage-tenure profiles during the Great Recession. In Section 4.4.3 we discuss other methods for identifying more financially constrained firms.

We consider a log wage equation according to (24). In equation (24) $w_{ij(t_0)t}$ and $T_{ij(t_0)t}$ are the earnings and tenure at time t of worker i, who was hired by firm j in t_0. We allow wages to be a non-parametric function of tenure, defined by tenure-specific coefficients β_s for $s = 0, 1, \ldots$. In equation (24) lev_{jt_0-1} is the leverage of firm j at time $t_0 - 1$, and X_{ijt_0} is a vector of firms' and workers characteristics at time t_0. We allow different wage-tenure profiles depending on firms leverage at the time of hire –whether it was above or below the median at $t_0 - 1$–, and on workers' and firms' observable characteristics at t_0. Wages also depend on a match-specific time invariant term, that we denote by μ_{ij}.

$$\log w_{ij(t_0)t} = \mu_{ij} + \sum_{s=0}^{S} \beta_s 1(T_{ij(t_0)t} = s) + \sum_{s=0}^{S} \gamma_s 1(T_{ij(t_0)t} = s) 1(lev_{jt_0-1} > \text{median}) + \sum_{s=0}^{S} \delta_s X'_{ijt_0} 1(T_{ij(t_0)t} = s) + u_{ij(t_0)t}, \tag{24}$$

Before discussing threats to identification, we take differences of (24) between t and t_0. On the left-hand side of (25) there is the cumulative wage growth between t and t_0 of worker i, who was hired by firm j in t_0. Since we are interested in differences in wage growth between firms with different leverage, our analysis focuses on the coefficients $\tilde{\gamma}_s = (\gamma_s - \gamma_0)$.[28]

[28]Note that permanent match-specific quality, as captured by the term μ_{ij}, is not a concern as it does not affect estimates of $\tilde{\gamma}_s$.

$$
\begin{aligned}
(\log w_{ij(t_0)t} - \log w_{ij(t_0)t_0}) = & \sum_{s=1}^{S} \tilde{\beta}_s 1(T_{ij(t_0)t} = s) \\
& + \sum_{s=1}^{S} \tilde{\gamma}_s 1(T_{ij(t_0)t} = s) 1(lev_{jt_0-1} > \text{median}) \\
& + \sum_{s=1}^{S} \tilde{\delta}_s 1(T_{ij(t_0)t} = s) X'_{ijt_0} + (u_{ij(t_0)t} - u_{ij(t_0)t_0}) \qquad (25)
\end{aligned}
$$

We restrict our attention to workers hired in January and whose first year of year-round employment at the firm was in $t_0 = 2009$. In this way, we compare the wage-tenure profiles of one cohort of workers hired during the Great Recession by different firms.[29] We focus on heterogeneous wage-tenure profiles within a relatively medium-term horizon of five years, where we expect to see more action, thus setting $S = 4$.

4.4.1 Identification

There are two main concerns associated with the estimation of $\{\tilde{\gamma}_s\}$ from equation (25). First, leverage is an endogenous variable that can be correlated with other firms' and workers' characteristics that influence future wage growth independently from leverage. Second, tenure is the outcome of endogenous decisions of workers, who may decide to change jobs, and firms, which can terminate a job. We now discuss how we address these concerns in detail.

Leverage at $t_0 - 1$ may be correlated with firms' and workers' observable characteristics that have an effect on the wage-tenure profile independently from leverage (e.g., leverage may be correlated with firms' size). To address this concern, we include firms' sector, total assets and value added per worker at $t_0 - 1$ in $X_{ij(t_0)t_0}$, thus controlling for sector, size and value added per worker specific tenure profiles. Similarly, we include dummy variables for workers' occupation, age and gender in $X_{ij(t_0)t_0}$, thus controlling for occupation, age and gender specific tenure profiles.

A similar concern is that leverage at $t_0 - 1$ may be correlated with firms' permanent unobservable characteristics that affect wage growth. For instance, some firms may have better access to credit and persistently sustain high leverage without being financially constrained.[30] To address this concern, we consider a specification similar to (25) where

[29] Pooling different cohorts of workers to compare post-Great Recession wage growth requires separately accounting for wage changes during the recession based on pre-existing tenure and experience. This is conceptually challenging and requires extensive data. Since data on workers' careers are available starting from 2005, tenure for all incumbent workers cannot be observed.

[30] A model assumption is that the collateral value of capital ξ is common to all firms. In practice different firms may have different values of ξ. Moreover, in the model financial constraints are occasionally binding, meaning that firms expect to ease the effects of the constraints over time. In practice some firms may always have high values of leverage, for instance due to "zombie lending".

we classify firms as financially constrained using within-firm variation in leverage, rather than pure cross-sectional variation (Ottonello and Winberry, 2020). More precisely, we allow the wage tenure profile to depend on whether leverage at t_0 was greater than the firms' own average, that is if $lev_{jt_0-1} > E[lev]_j$, where we calculate the average over the entire sample.

To further corroborate our results and address potential endogeneity issues, we perform a series of robustness exercises using alternative methods to identify more financially constrained firms, which will be discussed in more detail in the next section. Specifically, we employ a shift-share style research design to isolate local lending shocks based on variations in preexisting bank market shares and bank supply shifts (Greenstone, Mas, and Nguyen, 2020). This approach enables us to identify financially constrained firms without relying on firm-specific characteristics, thereby mitigating the endogeneity problems associated with using balance sheet data to derive proxies for financial constraints. Furthermore, our results remain robust even when using different measures of financial constraints derived from balance sheet data.

Comparing workers with the same tenure levels at different firms can induce bias in the coefficients $\{\tilde{\gamma}_s\}$ as workers' mobility and thus tenure is endogenous (e.g., better workers or higher-quality matches may correlate positively with lower quit rates, and workers' mobility decision may be correlated with firms' leverage). We address this issue by implementing two corrections that combine insights from Abraham and Farber (1987); Dustmann and Meghir (2005).

First, exploiting the nature of our exercise that aims to compare wage growth for workers in the first $S = 4$ years of tenure, we focus on a sample of workers hired at t_0 who then stay with the same firm for at least $S = 4$ years. This way, we compare workers with same *completed tenure*.[31] While comparing workers with same completed tenure controls for different quit rates, this step introduces sample selection. For example, highly levered firms may fire workers more often over time, keeping only the most productive ones.[32]

Second, we correct for sample selection using a common exclusion restriction (Dustmann and Meghir, 2005; Guiso, Pistaferri, and Schivardi, 2012), based on whether the *current* job is found following firm closure or a mass layoff at the *previous* job.[33] The idea is that displaced workers must start searching for a new job sampling from the uncondi-

[31] Since we are interested in tenure profiles over the first five years of tenure, the relevant completed tenure is within five years horizon, which we observe. If one was interested in estimating tenure profiles over an unbounded horizon, then a proxy for estimated completed tenure should be used as in Abraham and Farber (1987).

[32] A similar concern has to due with learning: highly levered firms may keep only workers that, over time, discovered to be the best fit for the firm. These effects are often ruled out in the empirical literature (Dustmann and Meghir, 2005), as the effects of tenure are difficult to interpret with learning about match quality.

[33] We define job displacement exactly as in Jacobson, LaLonde, and Sullivan (1993): we identify the workers who left their employer at the same time that the employer experienced a 30% or larger decline in employment.

tional distribution of match values, while those who moved voluntarily to the current firm did it because they improved their match value, (i.e., they sampled from the conditional distribution). Hence, the probability of being a mover out of the *current* job must be higher for the displaced worker than for the average worker.

4.4.2 Results

Table 2 reports the estimates of equation (25) using only the sample of job stayers. Since workers' mobility decision is endogenous, we present estimates with and without controlling for selection into the current job (Columns 2 and 3, respectively).

We report the probit estimates for completed tenure in Column 1, where the dependent variable is the probability that workers' completed tenure is greater or equal than S. These are used to construct an estimate of the inverse Mills ratio. The exclusion restriction in the mobility probit (a variable that affects the decision to move but does not affect wage growth) have the expected effect: being a displaced worker has a negative effect on completed tenure implying a higher probability of moving out of the current job. The instrument is statistically significant, suggesting that the estimates do not suffer from weak exclusion restriction problems. Further details about the mobility probit and the correction for sample selection are discussed in Appendix B.3.

We report estimates of the coefficients $\{\tilde{\gamma}_s\}$ in Columns 2 and 3. The estimated coefficients are positive, statistically significant, and increasing in tenure, meaning that highly levered firms offered steeper wage tenure profiles. This result aligns with our model prediction that during a credit tightening firms with high leverage offer a steeper wage-tenure profile to their workers. Our baseline measure of leverage is the ratio between firms' debt, measured as the sum of all financial debt and debt toward suppliers, divided by the firm's total assets. Adjusting for selection in Column 3 results in lower estimates of the coefficients $\{\tilde{\gamma}_s\}$, as expected. Moreover, the coefficients on the inverse Mills ratio become larger and statistically significant as tenure increases, suggesting that selection becomes more and more relevant over time.

4.4.3 Robustness

We perform several robustness exercises using alternative methods to classify firms as financially constrained. The results are reported in Table 3. In a first set of exercises, we explore alternative indicators of financial frictions derived from balance sheet data, using both cross-sectional and within-firm variation. We present additional evidence for these results by classifying firms as financially constrained based on heterogeneous fluctuations in local credit supply during the Great Recession.

First, we propose an alternative way to construct the leverage ratio b/k, where the denominator includes only financial debt, excluding debt to suppliers. We report results

Wage backloading of new hires			
	Probit	Cumulative wage growth	
	(1)	(2)	(3)
$1\left(lev_{jt_0-1} > \text{ median }\right)$	0.0486***		
	(0.0140)		
Tenure=1: $1\left(lev_{jt_0-1} > \text{ median }\right)$		0.0237***	0.0166***
		(0.0023)	(0.0019)
Tenure=2: $1\left(lev_{jt_0-1} > \text{ median }\right)$		0.0418***	0.0236***
		(0.0028)	(0.0023)
Tenure=3: $1\left(lev_{jt_0-1} > \text{ median }\right)$		0.0480***	0.0326***
		(0.0029)	(0.0024)
Tenure=4: $1\left(lev_{jt_0-1} > \text{ median }\right)$		0.0494***	0.0344***
		(0.0031)	(0.0026)
Displaced	−0.1553***		
	(.0372)		
Tenure=1: Inverse Mills ratio			-0.0005
			(0.0098)
Tenure=2: Inverse Mills ratio			0.0068
			(0.0118)
Tenure=3: Inverse Mills ratio			0.0293**
			(0.0123)
Tenure=4: Inverse Mills ratio			0.0334**
			(0.0132)
Workers' controls	Yes	Yes	Yes
Firms' controls	Yes	Yes	Yes
N. of workers	130,775	100,680	100,680
N	130,775	402,720	402,720

Table 2: Column (1) reports the results of the probit estimates of whether completed tenure of each worker-firm match is above or below S, using the entire sample of workers hired at t_0. Columns (2) and (3) report the estimates of equation (25) using the sample of workers who stayed at least 4 years. Robust standard errors in parenthesis. The superscripts ***,**, and * denote statistical significance at the 1, 5, and 10 percent level, respectively.

using cross-sectional variation in leverage and within-firm variation in leverage in Columns 1 and 2 respectively. As explained above, when using within-firm variation (Ottonello and Winberry, 2020) we replace the indicator function on whether firms' leverage at $t_0 - 1$ is above the median with an indicator function for whether firms' leverage at $t_0 - 1$ is above the firm's own average. In practice, the indicator function is replaced by $1(lev_{jt01} > E[lev]_j)$, where $E[lev]_j$ is the sample average of firm j. We find similar results using cross-sectional and within-firm variation in leverage. We interpret these results as evidence that our findings are not driven by permanent unobservable heterogeneity across firms.

Second, we consider firms with a high debt-to-output ratio to be more financially constrained, consistent with a view of earnings-based constraints as opposed to collateral constraints (Drechsel, 2023). In practice, firms' borrowing capacity might be limited by their earnings and not by the collateral value of their assets. Despite the fact that the Italian credit market relies more on collateralized borrowing than on this type of loan covenant, we find similar results. We measure firms' debt-to-output ratio as firms' financial debt divided by firms' value added. In this case, we also report results using cross-sectional variation and within-firm variation in debt-to-output ratio. In Appendix B.4 we present similar findings by classifying firms as more or less financially constrained based on their interest coverage ratio (Greenwald, 2019). These findings show that results are robust to alternative methods to classify firms as more or less financially constrained derived from balance sheet data.

Finally, we identify as more financially constrained those firms exposed to a more pronounced tightening of local credit conditions. To this end, we rely on a shift-share style research design to isolate local lending shocks using variation in preexisting bank market shares and bank supply shifts (Greenstone, Mas, and Nguyen, 2020). This approach allows us to identify more financially constrained firms without relying on firm-specific characteristics, thus addressing potential endogeneity problems associated with using balance sheet information to derive proxies for financial constraints.

We take an off-the shelf province-time index of credit supply from Barone, de Blasio, and Mocetti (2018), who use confidential information from Bank of Italy's Supervisory Report database.[34] We denote by S_{pt} a measure of the change in credit supply for province p in year t. This measure is the weighted average of bank-b-specific changes in outstanding loans δ_{bt} between year t and $t-1$, weighted by the market share W_{bp} of each bank b in

[34] Provinces are administrative units roughly comparable to a US county. According to the Italian Antitrust authority, the "relevant market" in banking for antitrust purposes is the province (Guiso, Pistaferri, and Schivardi, 2012). Italy is divided into 110 provinces.

Wage backloading of new hires: robustness					
	Leverage ratio (excluding suppliers)		Debt-to-output ratio		Local credit supply shock
	(1)	(2)	(3)	(4)	(5)
γ_1: tenure 1	0.0206***	0.0197***	0.0065***	0.0149***	0.0205***
	(0.0017)	(0.0018)	(0.0019)	(0.0019)	(0.0022)
γ_2: tenure 2	0.0320***	0.0402***	0.0156***	0.0259***	0.0346***
	(0.0021)	(0.0022)	(0.0022)	(0.0023)	(0.0027)
γ_3: tenure 3	0.0351***	0.0385***	0.0200***	0.0262***	0.0303***
	(0.0022)	(0.0023)	(0.0024)	(0.0028)	(0.0028)
γ_4: tenure 4	0.0323***	0.0345***	0.0204***	0.0284***	0.0310***
	(0.0024)	(0.0025)	(0.0026)	(0.0026)	(0.0030)
Within firm variation	No	Yes	No	Yes	
Inverse Mills ratio	Yes	Yes	Yes	Yes	Yes
Workers' controls	Yes	Yes	Yes	Yes	Yes
Firms' controls	Yes	Yes	Yes	Yes	Yes
N	402,720	402,720	402,720	402,720	395,000

Table 3: Estimates of the coefficients $\tilde{\gamma}_s$ from equation (25) obtained using different methods to classify firms as more or less financially constrained. Robust standard errors in parenthesis. The superscripts ***,**, and * denote statistical significance at the 1, 5, and 10 percent level, respectively.

province p before the crises.[35]

$$S_{pt} = \sum_b W_{bp} \widehat{\delta}_{bt} \tag{26}$$

We consider firms that operate in province p with an index of credit supply conditions S_{pt} below the mean as more financially constrained. The estimates in Column 5 of Table 3 indicate that wages grow faster in the first two years of tenure in firms experiencing a stronger deterioration in local credit conditions. These results, based on an exogenous measure of tight credit conditions, further corroborate the mechanism and address the remaining endogeneity issues associated with using balance sheet data to identify constrained firms. More details on this credit supply index and additional results are included in Appendix B.6.

One concern associated with the measurement of hourly earnings is that the estimates of equation (25) could still be influenced by adjustments in hours worked. Since we focus on full-time workers employed year-round in all empirical exercises, we expect hours to remain stable. However, firms may still adjust overtime hours for these workers. In

[35] In the baseline specification we use market shares in 2006. In Appendix B.6 we show that results are not sensitive to this choice.

Appendix B.5, we use data on average overtime hours at the firm level from the INVIND survey to show that differences in overtime hours between constrained and unconstrained firms are small and not statistically significant during and after the Great Recession and do not exhibit a similar dynamic pattern as that of wages.

In Appendix B.8 we exploit differences in wage flexibility across firms to propose descriptive evidence that links these wage adjustments to firms' investment decision: firms with greater wage flexibility backloaded wages more and experienced a smaller drop in investment during the Great Recession.

4.5 Incumbent workers during the Great Recession

Estimates of equation (25) provide evidence that more constrained firms temporarily backload wages of workers hired during the Great Recession. The model also predicts a slowdown in the wage growth of incumbent workers in more constrained firms when the recession hits. In this section, we document evidence supporting this prediction of the model.

We estimate an empirical specification similar to the one proposed in equation (25). However, in this case we consider a cohort of workers hired in $t_0 = 2007$ before the Great Recession and focus on the cumulative growth rate of wages between 2007 and 2009. On the left-hand side of equation (27) there is the cumulative wage growth from $t_0 = 2007$ to $t = 2009$ for worker i hired by firm j in t_0. On the right-hand side, the coefficient γ measures the additional wage growth for firms that are more financially constrained. As in the previous section, the vector X controls for workers' demographic characteristics by including dummies for occupation, age, gender. The vector X also controls for observable characteristics of firms, by including firm's size, value added per worker, sectoral dummies.

$$\log w_{ij(t_0)t} - \log w_{ij(t_0)t_0} = \beta + \gamma 1(lev_{jt-1} > \text{median}) + \delta X'_{ij(t_0)} + (u_{ij(t_0)t} - u_{ij(t_0)t_0}) \quad (27)$$

Estimates are reported in Table 4. As in the previous section, we report estimates using alternative methods to measure which firms are more financially constrained. The coefficient γ that measures the additional wage growth between 2007 and 2009 at more constrained firms is negative in all specifications. Column 1 reports estimates for the baseline specification, where we identify as more financially constrained firms with high leverage. The growth rate of wages between 2007 and 2009 was approximately 1.3 percentage points lower at firms with leverage above median. Column 2 and 3 reports estimates using within-firm variation in leverage and a shift-share credit supply index at the provincial level as explained in Section 4.4.

Wage Growth 2007-2009: incumbents			
	Leverage Ratio		Credit Shock
	(1)	(2)	(3)
γ	$-.013^{***}$	$-.019^{***}$	$-.009^{***}$
	(.002)	(.002)	(.003)
Within firm variation	No	Yes	
Inverse Mills ratio	Yes	Yes	Yes
Workers' controls	Yes	Yes	Yes
Firms' controls	Yes	Yes	Yes
Observations	421,430	421,430	402,541

Table 4: Estimates of the coefficient γ from equation (27) obtained using different methods to classify firms as more or less financially constrained. Robust standard errors in parenthesis. The superscripts ***,**, and * denote statistical significance at the 1, 5, and 10 percent level, respectively.

4.6 Estimates of pass-through coefficients

We estimate the pass-through coefficients of value added per worker to wages, following a well established methodology that built on Guiso, Pistaferri, and Schivardi (2005). Estimates of pass-through coefficients in the range between 5% and 15% have often been interpreted as evidence of some form of risk-sharing, or partial insurance, within long-term employment relationships.

In the empirical analysis, we isolate idiosyncratic changes in firms' value added per worker from fixed heterogeneity and changes common to all firms in the same sector. To this end, we construct residuals of log of value added per worker against firm-specific fixed effects and 2-digit NACE sector-year fixed effects. Then, we take the differences of the residuals that we denote by ε_{jt}. For each firm-worker match, we measure average monthly earnings in any given year. We focus on a sample of *stayers*, who work full-time and for 52 weeks in the same firm for at least two consecutive years.

We isolate idiosyncratic changes in workers' earnings from changes that can be attributed to demographics characteristics (gender-age-occupation fixed effects) or aggregate trends (year fixed effects). Then, we take first differences of the residuals, that we denote by ω_{jt}.[36]

Once we have constructed measures of the idiosyncratic components of earnings growth and growth in value added per worker, we estimate the pass-through of value added per worker to wages using standard techniques from Guiso, Pistaferri, and Schivardi (2005); Guiso and Pistaferri (2020). We define the pass-through coefficient as the follow-

[36] We winsorize ε_{jt} and ω_{jt} at the 1st and 99th percentile to remove for possibly extraordinary events that are not present in our model.

Estimates of pass-through coefficients			
	(1)	(2)	(3)
$\Delta\varepsilon_{jt}$	0.0612***		
	(0.0004)		
$\Delta\varepsilon_{jt}\times$ 1(Below median leverage)		0.0474***	0.0414***
		(0.0004)	(0.0005)
$\Delta\varepsilon_{jt}\times$ 1(Above median leverage)		0.0680***	0.0760***
		(0.0006)	(0.0005)
Firms' controls	No	No	Yes
N	11,052,040	11,052,040	11,052,040

Table 5: Estimates of the pass-through coefficient of firms' value added per worker to wages, obtained using the instrumental variable estimator defined in (28). The first column reports the average pass-through. In the other columns we split the sample and report estimates for firms with leverage above and below median. Robust standard errors in parentheses. The superscripts ***,**, and * denote statistical significance at the 1, 5, and 10 percent level, respectively.

ing moment of the data:

$$\mathcal{P} = \frac{Cov\left(\Delta w_{ijt}, \sum_{s=-1}^{1} \Delta\varepsilon_{jt+s}\right)}{Cov\left(\Delta\varepsilon_{jt}, \sum_{s=-1}^{1} \Delta\varepsilon_{jt+s}\right)} \tag{28}$$

which corresponds to the instrumental variable regression of Δw_{ijt} on $\Delta\varepsilon_{jt}$, using $(\varepsilon_{jt+1} - \varepsilon_{jt-2})$ as an instrument. The instrumental variable strategy filters out the effect of purely transitory shocks, which we interpret as measurement error. In a model with permanent productivity shocks and static pass-through as Guiso, Pistaferri, and Schivardi (2005), this estimator recovers the true pass-through of shocks to the persistent component of firms' idiosyncratic productivity to workers' earnings.

In our model, the pass-through coefficient is not static –as wages adjust dynamically in response to shocks– and productivity shocks are not permanent. Consequently, in our model the pass-through coefficient of value added per worker to wages is different from the pass-through of idiosyncratic productivity shocks to wages. We regard these pass-through coefficients of value added per worker to wages as crucial moments of the data that measure the degree of risk-sharing between firms and workers, which we use to discipline and validate our model. We estimate an average pass-through coefficient of 6.1%, as reported in Column 1 of Table 5, in line with the existing literature. Using Italian data from 1982 to 1994 Guiso, Pistaferri, and Schivardi (2005) finds an average pass-through coefficient of 6.8%.

We proceed by estimating how the pass-through coefficient varies with firms' leverage. We split firms according to whether at $t-1$ their leverage was above or below the median in the leverage distribution of that year, and estimate the pass-through coefficient for each sub-sample. Our baseline measure of leverage is the ratio between firms' debt, measured as the sum of all financial debt and debt toward suppliers, divided by the firm's total assets. The results are reported in Column 2 of Table 5.[37] Firms with leverage above median have a pass-through coefficient of 6.8%, that is almost 1.5 times larger than the estimated pass-through for firms having leverage below median.

As we discussed in Section 4.4, leverage can be correlated with other firms' observable characteristics that affects the pass-through coefficient independently from leverage.[38] In Column 3 we present results obtained by first residualizing leverage at $t-1$ against log assets and log value added per worker at $t-1$, and then categorizing firms based on whether this residualized measure of leverage is below or above the median. Controlling for firms' observable characteristics, firms with leverage above median have a pass-through coefficient of 7.6%, that is almost 1.8 times larger than the estimated pass-through for firms having leverage below median.

In Appendix B.7 we illustrate how the results presented in Table 5 are robust to using alternative methods to classify firms as financially constrained.

5 Quantitative analysis

We present the quantitative model starting from the calibration of model parameters. Then, we assess the ability of the quantitative model to reproduce salient features of the data in Section 5.2. Sections 5.3 and 5.4 present the main quantitative results aimed at quantifying the importance of dynamic wage contracts for the propagation of financial shocks and for the effects of stabilization policies. We solve the model using techniques from Krusell and Smith (1997), including the price q as a state variable in the entrepreneurs' problems and approximating a forecasting rule for future prices. Details are described in Appendix C.

5.1 Calibration

We begin by describing how we choose parameters for our quantitative analysis. We interpret one period in the model as one quarter. The firm-level production function and the matching technology are both Cobb-Douglas and are described in equations (29), (30), where B is a constant that measures matching efficiency, η is the matching function

[37] Results are robust to considering alternative measures of leverage.

[38] Correlation of firms' leverage with workers' observable characteristics is not a source of concern here, as ω_{ijt} is the idiosyncratic component of earnings that is orthogonal to workers' observable characteristics.

Assigned parameters			
Parameter	Intuition		Value
β	Discount factor		0.99
δ	Depreciation rate		0.025
α	Capital share		0.3
η	Matching function elasticity		0.5
σ_E	RA coefficient of entrepreneurs		1
$P(\xi_L\|\xi_H)$	Probability of financial recession		0.01
$P(\xi_L\|\xi_L)$	Persistence of financial recession		0.8
ϕ	Separation probability		0.024
x	Share of time spent searching for a job		0.05
Externally calibrated parameters			
Parameter	Intuition		Estimated value
ρ	Persistence of idiosyncratic productivity z		0.96
$\sigma(\varepsilon)$	Std. deviation of innovations in z		0.07

Internally calibrated parameters				
Parameter	Value	Moment	Model	Target
ξ_L	0.33	Drop of corporate debt in 2008	-14%	-14%
ξ_H	0.87	99p of leverage distribution	0.87	0.87
σ_W	11	Pass-through of VA/worker to wages	0.06	0.06
B	0.5	Job finding rate	0.30	0.30
$\bar{b}$	0.15	Unemployment rate	8.1%	8.1%

Table 6: Values for all model's parameters.

elasticity and α is the production function elasticity.

$$f(k,\ell) = k^{\alpha}\ell^{1-\alpha} \tag{29}$$

$$m(v,s) = Bv^{1-\eta}s^{\eta} \tag{30}$$

Nine parameters are assigned and listed in Table 6. We set the common discount factor β equal to 0.99, the depreciation rate of capital equal to 0.025, and the elasticity of the Cobb-Douglas production function equal to 0.3, as these are standard values in macroeconomics. We follow Ljungqvist and Sargent (2017) in setting the elasticity of the matching function η equal to 0.5, based on estimates for several EU countries reviewed in Petrongolo and Pissarides (2001). We assume that entrepreneurs have log-utility, that is $\sigma_E = 1$, following a large body of work that studied the effects of financial frictions in models with heterogeneity, such as Midrigan and Xu (2014), Moll (2014), Kiyotaki and Moore (2019).[39] We set $P(\xi_H|\xi_H) = 0.99$ to be consistent with the notion that financial crises are rare events in advanced economies.We set $P(\xi_L|\xi_L) = 0.8$, meaning that the average duration of a recession in the model is five quarters, which corresponds to the

[39] This assumption is not far from empirical evidence. Estimates from Herranz, Krasa, and Villamil (2015) imply a median relative risk aversion coefficient of entrepreneurs of 1.5, obtained using a sample of small firms in the US.

length of the Great Recession in Italy according to the OECD based recession indicators. Finally, we set the probability of separation in the model equal to 0.024 to match the separation rate measured in D'Amuri et al. (2022) for Italy.[40] We set the disutility cost of searching for a job in line with evidence on the amount of time spent searching for a job (Manning, 2011; Krueger and Mueller, 2010). In practice, we set x equal to 0.05, which means that agents must forego 5% of the value of home production when looking for a job.

The remaining parameters are estimated in two separate exercises. We recover the parameters that discipline the stochastic process for firms' idiosyncratic productivity, the persistence ρ and the standard deviation $\sigma(\varepsilon)$, by estimating a stochastic process for firms' productivity from balance sheet data. We use a GMM estimator that filters out fixed heterogeneity across firms and purely transitory productivity shocks, as none of them are present in the model. Details are discussed in Appendix B.9. The remaining five parameters $(\xi_H, \xi_L, B, \bar{b}, \sigma_W)$ are chosen simultaneously so that the model matches a set of moments of interest. Below, we describe the targeted moments and discuss heuristically which model parameters they help us discipline.

We pin down the collateral value of capital ξ_L during financial crises by targeting the observed 14% drop in aggregate debt of the non-financial sector in 2008[41], following the same calibration strategy proposed in Khan and Thomas (2013). We set the collateral value of capital during normal times ξ_H, that is the maximum leverage b/k that a firm can have, as to match the 99th percentile of firms' leverage distribution. Motivated by the analytical results illustrated in Section 3, that is wages co-move with entrepreneurs' consumption according to the ratio of the two relative risk aversion coefficients, we set the relative risk aversion coefficient of workers in order to match the average pass-through of value added per worker to wages that we estimated in Section 4. We estimate the value of home production $\bar{b}$ and the efficiency of the matching function B following a calibration strategy similar to Shimer (2005). We target an average unemployment rate of 8. 1%, which was the average unemployment rate in Italy for the years before 2008, and an average job finding rate equal to 30%, consistent with empirical evidence from D'Amuri et al. (2022), Cingano and Rosolia (2012).

[40] Using microdata from the Labour Force Survey, D'Amuri et al. (2022) find a quarterly EU rate (employment to unemployment) of 1.2% and a quarterly EN rate (employment to out of labor force) of 1.2% for men aged 35-55, that implies a quarterly separation rate equal to 2.4%. We focus on males between 35-55 as the impact of fertility, schooling, and retirement decision on the EN rate is negligible, as these features are not present in the model.

[41] The outstanding amount of total debt securities in non-financial corporations sector was 100 billions in Q1 of 2008 and 87 billions in Q4 of 2008.

5.2 Model vs Data

The quantitative model can match several untargeted moments describing wage dynamics, leverage distribution, and investment dynamics. The first panel of Table 7 shows that the model can quantitatively replicate the heterogeneity in wage dynamics for firms with different leverage that we documented in Section 4.6. As in the data, the pass-through of value added worker to wages is higher for firms with leverage above the median, and the magnitudes are comparable. Also, we estimate the wage tenure-profile for workers hired in recession by firms with leverage above and below the median. We find that the wages of workers hired by highly leveraged firms during a recession increase by approximately 2 percentage points after one year of tenure and 3 percentage points after four years of tenure, consistent with the empirical evidence presented in Section 4.4.

The model generates a cross-sectional distribution of firms' leverage similar to the data. Panel B in Table 7 reports the three quartiles of the leverage distribution, that are remarkably similar between the model and the data, despite targeting only the 99th percentile. Note that in normal times only few firms face a binding collateral constraint, as the 75th percentile of the leverage distribution is substantially below ξ_H. We measure the average standard deviation of investment rates (i/k) in the data using a balanced panel of firms, finding the same value documented in Cooper and Haltiwanger (2006), which is very close to the value implied by the model.[42]

We inspect the ability of our model to reproduce the dynamics of macroeconomic aggregates during the 2008 recession in Italy. Panel C in Table 7 reports the drop in aggregate output, employment, total factor productivity, and investment from peak to trough during the first year of recession. The model has been calibrated to match a drop in aggregate debt of 14%, and it accounts for a substantial share of the observed drop in aggregate output, consistent with the view that financial shocks played a key role in the Great Recession.

The model also implies a substantial drop in aggregate employment, investment, and total factor productivity. The dynamics of employment and capital depend on a direct effect and a general equilibrium effect.

A drop in the collateral value of capital ξ leads to a contraction in debt and investment for entrepreneurs with low net-worth that are not able to self-finance themselves. The contraction in aggregate debt generates excess savings in the economy that increases the price of bonds q. At the same time, an increase in q induces wealthy unconstrained entrepreneurs to substitute away from risk free bonds and to invest more in physical capital.[43]

[42] An alternative calibration strategy for the productivity process would have been to estimate the standard deviation of innovation to productivity by targeting the standard deviation of investment rates, as in Khan and Thomas (2013). We rather estimate a productivity process externally and we keep this moment for the validation.

[43] This heterogeneous effects of an aggregate financial shock on firms' investment decision is similar to

Panel A: Wage dynamics				
	Pass-through coefficients × 100		Additional wage growth at levered firms	
	Leverage < median	Leverage > median	1y horizon	4y horizon
Model	4.1	7.1	1.9pp	3.0pp
Data	4.1	7.6	1.7pp	3.4pp
Panel B: Investment and leverage				
	p25 leverage	p50 leverage	p75 leverage	Investment volatility
Model	0.20	0.46	0.63	0.30
Data	0.25	0.43	0.70	0.33
Panel C: Macroeconomic effects of a financial shock				
	Drop in aggregate variables, first year of recession			
	$\Delta \log(Y_t)$	$\Delta \log(N_t)$	$\Delta \log(I_t)$	$\Delta \log(A_t)$
Model	-4.7 %	-2.9%	-24 %	-3.6 %
Data	-7.1 %	-3.4%	-12 %	.
Panel D: Distribution of wage adjustments				
	Skewness		Kelley's skewness	
	Recession	Normal times	Recession	Normal times
Model	1.1	2.5	-0.47	- 0.02
Data	0.20	0.71	0.01	0.20

Table 7: Panel A reports moments associated to wage dynamics in the model and empirical findings from Section 4. Panel B reports moments for the distribution of leverage and investment dynamics in the model and in the data. Panel C reports the drop of macroeconomics aggregates from peak to trough in the first year of the recession in the data and according to the impulse response functions of the model. Panel D reports the skewness of the wage adjustment distribution in the model and in the data (Adamopoulou et al., 2016).

The response of aggregate employment is disciplined by the job creation decision of unmatched entrepreneurs. When ξ drops, the surplus of a match $J - V$ falls for entrepreneurs with low net worth. As a result, they either post a lower promised value W – with a drop in the vacancy filling rate $\lambda_f(\theta)$– or they decide not to open a vacancy at all, leading to a drop in aggregate employment. In general equilibrium, the price of bonds increases, fostering job creation of unconstrained entrepreneurs. In fact, entrepreneurs with high net worth experience an increase in the match surplus $J - V$, as investment in capital becomes relatively more attractive than investing in risk-free bonds.

While the direct effect dominates, the coexistence of these two countervailing channels affects allocative efficiency. Because of the direct effect, constrained entrepreneurs with a high marginal product of capital are forced to reduce their investment, while wealthy entrepreneurs with lower marginal product of capital invest more. This reallocation of capital towards unconstrained entrepreneurs leads to a drop in aggregate productivity A, as defined in equation (11).

The model is consistent with not only the evidence on the heterogeneous wage dynamics presented in Section 4, but also with stylized facts on the aggregate wage dynamics. The model implies a modest cyclicality of the average wage in recession, that drops by 0.3%. The modest decline in the average wage masks substantial heterogeneity in the cross-section: firms strongly impacted by the credit tightening cut their wages substantially, while unconstrained firms expand and pay higher wages. In the aggregate, this firm-level heterogeneity in wage adjustments implies that the skewness of the wage adjustment distribution is lower in recessions than in booms. This characteristic of our model is consistent with recent evidence that the skewness of the wage adjustment distribution changes during recession (Adamopoulou et al., 2016), as reported in Panel D of Table 7.[44] The model is also consistent with evidence from Gertler, Huckfeldt, and Trigari (2020), Grigsby, Hurst, and Yildirmaz (2021) that the wages of new hires are as cyclical as the wages of incumbents, and with evidence from Kudlyak (2014) and Basu and House (2016) that the average user cost of labor is substantially more cyclical than the average wage. We provide more details in Appendix C.5.

5.3 The macroeconomic implications of dynamic wage contracts

Dynamic wage contracts, meaning the ability of firms to adjust wages over time and in response to shocks, play an important role when the economy is hit by an aggregate financial shock. In order to quantitatively evaluate this channel, we compare our model

the one illustrated by Khan and Thomas (2013).

[44] The model is consistent with evidence that the skewness of the wage adjustment distribution dropped substantially during recessions. The values of such skewness differ between model and data because several other characteristics that affect wage adjustments, such as human capital, are not present in the model.

to a counterfactual economy where firms cannot optimally adjust wages.

There are two fundamental assumptions in the model that allow firms to adjust wages over time and in response to shocks: employment relationships are long-term in nature and firms can commit to future wages. Indeed, for firms to be able to backload wage payments they must stay matched with the same workers for more than one period, and they must be able to implicitly promise higher future wages to their employees. To quantitatively assess the importance of dynamic wage contracts during financial crises, we relax the former assumption on firms' commitment, as this exercise can be done without changing the economic environment.[45]

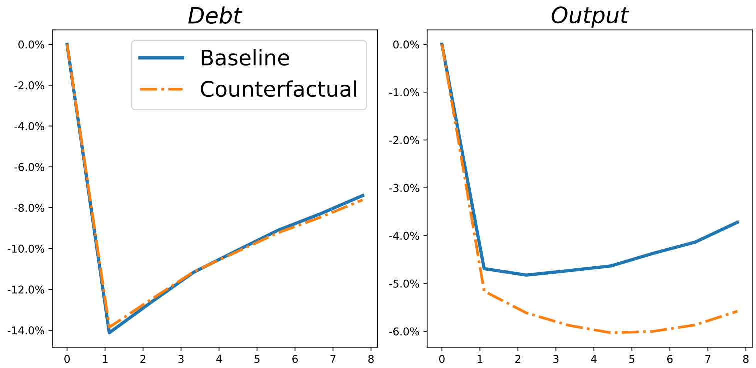
We propose a counterfactual economy that has two features of a spot labor market: firms cannot adjust the timing of wage payments over the length of an employment relationship, and the allocative wage for job creation is only the current wage w_t. These features are obtained by assuming that entrepreneurs can commit only to a wage for the first period after matching with a worker. More precisely, if an entrepreneur hires a worker in period t, we assume that the entrepreneur can commit only to a wage w_{t+1}, that is, for the first period in which the match is productive. Consistently with the assumption of no commitment on the firm's side, we also assume that workers cannot commit to an employment relationship with a given entrepreneur. As a result, entrepreneurs would pay workers their outside option $\bar{b}$ for any period $s > t + 1$, as this is a dominant strategy compared to any wages above $\bar{b}$. As in the baseline model, firms compete for workers in the expected utility W of a match, but do so by choosing only w_t. Therefore, the main difference from the baseline model is that firms cannot choose how to deliver the utility W over time and in response to shocks by adjusting the wages.

Figure 2 plots the impulse response function of macroeconomic aggregates to an aggregate financial shock in the baseline model and in the counterfactual economy with no commitment. The two economies display the same drop in aggregate debt.[46] However, in the counterfactual economy output is more than one percentage point lower compared to the baseline economy, meaning that dynamic wage contracts substantially mitigate the effects of an aggregate financial shock on output.

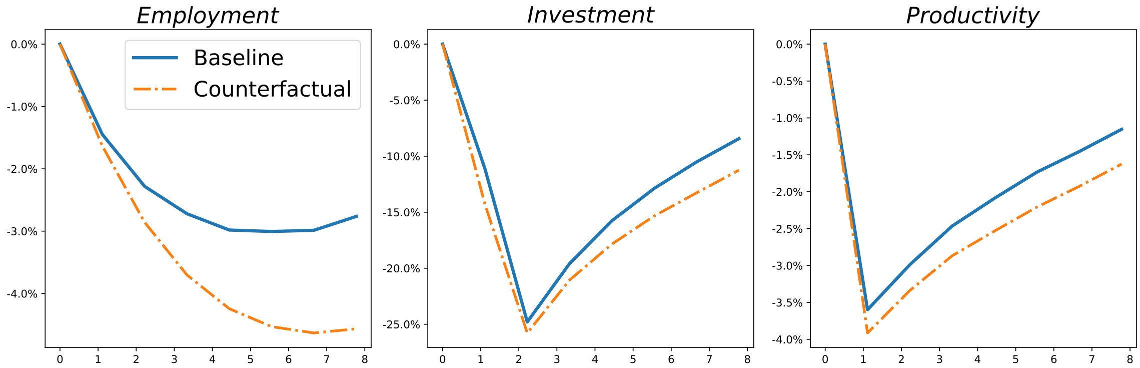
The differential response of output is primarily driven by differences in aggregate employment and productivity. In the baseline economy, firms optimally adjust wages to ease the effects of the credit tightening. As a result, aggregate employment and investment fall less in the baseline economy, as illustrated in Figure 2. In other words, in the baseline economy financially constrained entrepreneurs invest more than in the counterfactual

[45]On the other hand, relaxing the assumption of long-term employment relationships would substantially change the economic environment, thus limiting the extent of the quantitative exercise to capture only the role of dynamic wage contracts, which is the main channel we aim to isolate.

[46]We re-calibrated the value of ξ_L in the counterfactual economy as to obtain the same drop in aggregate debt, as we interpret a drop in aggregate debt, rather than a lower value of ξ_L *per se*, is the primitive shock propelling a financial recession.



(a) Aggregate debt and output



(b) Aggregate employment, investment, and productivity

Figure 2: Impulse response functions for aggregate debt (D), output (Y), employment (N), investment (I) and productivity (A) in response to an aggregate financial shock. The solid line plots impulse response functions in the baseline economy, and the dashed line plots impulse response functions in the counterfactual economy with no commitment. We compute $2 \times M$ simulations of length T. We draw M sequences of uniform random numbers that we use to simulate realizations of ξ. In the first M simulations we set $\xi = \xi_L$ at $T - 10$. The IRFs are computed taking the difference in logs between the first and second set of simulations from $T - 10$ to T, averaging across M.

economy. This implies that in general equilibrium unconstrained entrepreneurs invest *less* in the baseline economy, since the opportunity cost of capital q is greater. As a result, the impulse response functions of aggregate investment do not differ substantially between the two economies. However, this general equilibrium effect implies that in the baseline economy aggregate productivity is higher, as capital is reallocated from unconstrained entrepreneurs with low marginal product of capital to constrained entrepreneurs with high marginal product of capital. This reallocation implies that the drop in aggregate productivity is less pronounced in the baseline economy, as shown in Figure 2.

In Appendix C we consider a comparative static exercise where we increase the relative risk aversion coefficient of workers, thus decreasing firms' ability to adjust wages, and we find results similar to Figure 2.

5.4 Wage adjustments and stabilization policies

We now turn to study the effectiveness of stabilization policies aimed at reducing input costs, in light of our findings that firms can optimally adjust the timing of wage payments over long-term employment relationships to reduce the cost of labor. We consider two broad types of policies: payroll subsidies and investment subsidies.

Payroll subsidies have been implemented by several OECD countries after the Great Recession in the form of payroll tax cuts. In some cases the tax cut was small and applied to all employees, but more often it has been generous and it applied only to new hires. In the US, the social security contribution for workers hired from unemployment has been set to zero as part of the HIRE Act, leading to a 6.2 percentage points cut. The contribution has been even more generous in some European countries, such as Ireland and Portugal, where the cost of the employment contribution has been set to zero in 2010, leading to a 10 percentage points cut.[47] These policies are often motivated by the presence of wage rigidity (Bils and Klenow, 2009) and targeted towards new hires based on the idea that the cost of incumbent workers is infra-marginal and thus not allocative. We focus on payroll subsidies for new hires, as these policies have been more used in practice and there is a large consensus on their potential positive effects.[48]

Investment subsidies have often been implemented to foster recoveries in downturns. In practice, these subsidies are often implemented using accelerated depreciation schemes, that allow firms to deduct a large share of their investment from taxes immediately. Both in 2003 and in 2008 the United States introduced a 50 percent bonus depreciation, giving firms the possibility to immediately deduct 50 percent of investment purchases and then depreciate the remaining 50 percent under standard depreciation schedules. According to House and Shapiro (2008) this policy was equivalent to an investment subsidy ranging between 0.5% and 4.5%, depending on the recovery period of the investment good and the nominal interest rate. To facilitate the comparison between payroll subsidies and investment subsidies, we focus on investment subsidies targeted to newly created matches.

How effective are these policies when firms can optimally adjust the timing of wage payments over long-term employment relationships? To answer these questions we compare the effects of these policies between our model and the counterfactual economy described in Section 5.3 where firms cannot adjust wages over time. We show that a payroll subsidy on new hires is not as effective as in the counterfactual economy because it crowds out the incentives of firms to backload wages and its effectiveness is lower when firms already pay lower wages during recessions. On the other hand, an investment sub-

[47] See OECD (2010) for a detailed discussion of payroll tax cuts and hiring credit measures after 2008.

[48] As illustrated by Schoefer (2021), in a model with financial constraints, broad-based subsidies applied to incumbent workers would also increase hiring and investment because the cost of incumbent workers is not infra-marginal. Another rational for broad-based subsidies is to prevent firms from firing workers during downturns and to prevent a spike in unemployment. Since in our model separation is exogenous and matches are one-to-one, we focus on policies targeted to newly created matches.

sidy makes firms want to invest more, which spur firms' incentives to backload wages as the current marginal value of a dollar in the firm increases relative to the future. As a result, firms make the wage tenure profiles steeper in response to the policy making more resources available for investment and amplifying the initial stimulus.

5.4.1 Modeling payroll and investment subsidies

We introduce payroll and investment subsidies in the baseline model. The government sets payroll subsidies, investment subsidies, lump sum taxes, and government debt. The subsidies $\tau_N(\xi), \tau_I(\xi)$ depend on the realization of the aggregate shock, with $\tau_N(\xi_L) > 0, \tau_I(\xi_L) > 0$ and $\tau_N(\xi_L) = 0, \tau_I(\xi_L) = 0$, meaning that there is a temporary subsidy in recession and no taxes or subsidies in normal times.[49] While the government is restricted to running a balanced budget fiscal policy in the long run, we allow for short-run debt-financed government expenditure. This means the government can raise funds using public debt and lump-sum taxes. Denote by G government expenditure on payroll and investment subsidies. The budget constraint of the government is described by equation (31).[50] We parameterize the persistence of government debt by ρ_B. Given the law of motion of government debt (32), lump-sum taxes are set period by period to satisfy the budget constraint (31). We assume that lump-sum taxes are levied on entrepreneurs and employed workers.

$$qB' = G + T + B \tag{31}$$

$$B' = \rho_B(B + G) \tag{32}$$

The problem of matched entrepreneurs in an economy with subsidies is defined in equation (33). Due to investment subsidies, the value of a matched entrepreneur now depends explicitly on the current capital stock k. Since we focus on policies targeted to newly created matches, the value of a matched entrepreneur depends on whether he is eligible for the subsidy ($e = 1$) or not ($e = 0$). Newly created matches are always eligible for the subsidy, and an entrepreneur remains eligible if $\xi' = \xi_L$. The budget constraint and the law of motion of net worth are modified to account for subsidies and lump sum transfers T. Both policies transfer resources to possibly constrained entrepreneurs by reducing the cost of inputs and at the same time provide incentives for job creation and investment. More details on the economy with subsidies are provided in Appendix C.2.

[49] An alternative way to perform this exercise is to assume there are payroll taxes in normal times, that is $\tau_N(\xi_H) < 0$, and a payroll tax cut, rather than a payroll subsidy, in recession. This exercise would be country-specific. We choose a more stylized approach to make a broad argument on the effects of such policies. A logic applies to investment subsidies.

[50] We allow the government to issue public debt for two reasons. First, an increase in government expenditure during economic downturns is often funded by government debt and not by a contemporaneous increase in taxes. Second, the use of government debt to foster economic recovery can play an important role during credit tightening, when private credit markets are disrupted

$$J(m,W,z,k,e,S) = \max_{\substack{c^e,b',k',i,m'(z',\xi')\\ w'(z',\xi'),W'(z',\xi')}} \Big\{ v(c^e) + \beta(1-\phi)\mathbb{E}\left[J(m'(z',\xi'),W'(z',\xi'),z',k',e',S')|z,S\right]$$
$$+\beta\phi\mathbb{E}\left[V(m'(z',\xi'),z',S')|z,S\right]\Big\} \qquad (33)$$

(Budget constraint : λ^e) $\quad c^e + i[1-e\tau_I] \leq m - k(1-\delta) + q(s_0)b'$

(Capital : I) $\quad k' \leq i + (1-\delta)k$

(Net worth : $\eta(z',\xi')$) $\quad m'(z',\xi') \leq z'f(k') + (1-\delta)k' - w'(z',\xi')[1-e\tau_N] - b' + T'(\xi',S')$

(Collateral constraint : μ) $\quad b' \leq \xi k'$

(Promise keeping : γ) $\quad W \leq \mathbb{E}\Big[u(w'(z',\xi') + T'(\xi',S')) + \beta(1-\phi)W'(z',\xi') + \beta\phi\mathcal{U}(S'')|z,S\Big]$

5.4.2 The effects of payroll subsidies

We consider first the effects of payroll subsidies. When the HIRE act was passed in the United States, the former Chief Economist at the Treasury Alan Krueger said that the HIRE act "provides an incentive for private-sector employers to hire new workers sooner than they otherwise would". The ability of these policies to stimulate employment depends crucially on how they reduce the cost of labor. The cost of labor in present discounted value terms, accounting for payroll subsidies is:

$$PDV_t = \mathbb{E}\left[\sum_{s=t}^{\infty} \underbrace{[1-\tau(\xi_s)] \times w_{s+1}(z_{s+1},\xi_{s+1})}_{\text{flow cost of labor}} \times \underbrace{[\beta(1-\phi)]^{(s-t)}\frac{\eta_{s+1}(z_{s+1},\xi_{s+1})}{v'(c^e_t)}}_{\text{SDF of entrepr.}}\right] \qquad (34)$$

Crucially, temporary payroll subsidies lower the cost of labor for new hires according to the share of PDV_t that is paid during the recession, that is, as long as the subsidy is in place. As the stochastic discount factor of entrepreneurs increases substantially during a credit tightening, the share of PDV_t paid in recession is large. This implies that temporary payroll subsidies can have large effects on employment during financial crises[51] On the other hand, the share of the flow cost of labor that is paid in recession depends on the solution of the dynamic contracting problem. To highlight this channel,

[51] This is in line with evidence from Saez, Schoefer, and Seim (2019), finding that payroll subsidies are more effective on financially constrained firms.

we study the effects of payroll subsidies in the baseline economy and in the counterfactual economy with no commitment that we presented in Section 5.3.

In the model, firms backload the wages of new hires after an aggregate financial shock. When wages are backloaded, the share of PDV_t that is paid in recession, that is as long as the subsidy is in place, is lower because the flow cost of labor is lower. Consequently, one should expect temporary payroll subsidies to be less effective in lowering the present discounted value of wage payments in the baseline model compared to a counterfactual economy with no dynamic wage contracts.[52]

Temporary payroll subsidies also have an effect on wages, as subsidies distort the risk-sharing condition between entrepreneurs and workers, as illustrated in equation (35). Indeed, the policy provides incentives to pay higher wages in recession because it is less costly to provide utility to workers when wages are subsidized. In this sense, in our model entrepreneurs adjust wages in response to a payroll subsidy to increase the flow cost of labor and reducing resources available for investment.

$$\eta(z',\xi')[1-\tau(\xi')] = \gamma u'\left(w'\left(z',\xi'\right)\right) \tag{35}$$

Table 8 reports the effects of a payroll subsidy on new hires equal to 6 percentage points, such as the one introduced in 2010 in the United States as part of the HIRE Act. The effect on aggregate output is substantially lower in the economy with dynamic wage contracts, both on impact and cumulatively one year after the beginning of the recession. Payroll subsidies for new hires are not as effective as they would be in a model in which firms cannot adjust wages over long-term employment relationships. Intuitively, firms' optimal wage adjustments and payroll subsidies are substitute to each other, as they both aim to reduce the cost of labor.

5.4.3 The effects of investment subsidies

We now turn to studying the effects of investment subsidies. Evidence from House and Shapiro (2008), Zwick and Mahon (2017) shows that firms respond strongly to these policies and that financial frictions can amplify the investment response. Intuitively, constrained firms value future cash flows with high effective discount rates, which amplify the perceived value of bonus incentives because the difference in todays tax benefits dwarfs the present value comparison that matters in frictionless models. We illustrate the differential effects of these policies on constrained and unconstrained firms using the optimality conditions implied by (33). To understand this heterogeneous response, we focus on whether it affects the intertemporal consumption margin, and thus wages through the risk sharing condition.

[52]Similarly, a consequence of wage backloading is also that the increase of the stochastic discount factor of entrepreneurs during recession is less pronounced.

Panel A: Payroll subsidies						
	Effect on impact: $\Delta \log X_t$			Cumulative effect: $\sum_{s=0}^{3} \Delta \log X_{t+s}$		
	Y	N	I	Y	N	I
Baseline model	+0.1 %	+0.1%	+1.6 %	+1.1 %	+1.3%	+5.9 %
Counterfactual	+0.2 %	+0.2%	+3.3 %	+1.4 %	+1.3%	+11.2 %
Panel B: Investment subsidies						
	Effect on impact: $\Delta \log X_t$			Cumulative effect: $\sum_{s=0}^{3} \Delta \log X_{t+s}$		
	Y	N	I	Y	N	I
Baseline model	+1.3 %	+1.5%	+10 %	+6.3 %	+7.4%	+32 %
Counterfactual	+0.8 %	+0.8%	+4.2 %	+5.1 %	+5.7%	+17 %

Table 8: Macroeconomic effects of a temporary payroll and investment subsidy on newly created matches. Panel A reports results for an economy with only payroll subsidies $\tau_L(\xi_L) = 0.06$, while Panel B reports results for an economy with only investment subsidies $\tau_I(\xi_L) = 0.016$. The table reports differences in output, employment and investment between the economy with subsidies and the economy without. Differences are reported on impact and cumulatively at one year horizon. The first line of each panel reports results for the baseline model, while the second line for the counterfactual model. We set $\rho_B = 0.9$.

When the constraint does not bind, the intertemporal consumption decision is unaffected by the investment subsidy, and is characterized by the Euler equation for bonds in equation (36). An investment subsidy makes capital more attractive, and this affects the optimal allocation of resources between debt and capital for unconstrained entrepreneurs. However, as long as entrepreneurs are not financially constrained, the subsidy does not affect the intertemporal consumption decision and the Euler equation for bonds (36) is identical to the in the model without taxes. Through the risk-sharing condition this means that also the intertemporal path of wages is not affected by the subsidy.

$$qv'(c^e) = \mathbb{E}[\eta(z', \xi')] \tag{36}$$

However, when the constraint binds the investment subsidy affects the intertemporal consumption decision. By combining the optimality conditions for bonds and capital, we obtain equation (37). An investment subsidy makes capital more attractive, but since constrained entrepreneurs cannot substitute away from bonds to invest more in capital, as the choice of bonds is not interior, the only way they can invest more in capital is by reducing their current consumption. As a result, the investment subsidy affects the intertemporal consumption decision for constrained firms by increasing the current marginal value of a dollar relative to the future. As entrepreneurs make their own consumption profile steeper in response to the subsidy, the risk sharing condition implies that they also make the wage tenure profile of their workers steeper. In this sense, in our model entrepreneurs adjust wages in response to an investment subsidy to free more resources for investment, thus amplifying the initial stimulus.

$$v'(c^e)[1-q\xi] < \mathbb{E}\Big[\eta(z',\xi')(z'f(k')+1-\delta-\xi)\Big] + \underbrace{\Big(\tau v'(c^e) - (1-\delta)\mathbb{E}[\eta(z',\xi')\tau(\xi')]\Big)}_{>0} \tag{37}$$

Table 8 reports the effects of an investment subsidy equal to 1. 6%, which according to House and Shapiro (2008) mimics the effects of a bonus depreciation of 50%.[53] The effect on aggregate output is substantially larger in the economy with dynamic wage contracts. Investment subsidies are more effective than they would be in a model in which firms cannot adjust wages in long-term employment relationships. Intuitively, firms' optimal wage adjustments and investment subsidies are complement to each other: when the policy reduce the cost of investment by making capital more attractive, firms respond to the policy by backloading wages in a way to free more resources to invest in capital thus amplifying the effects of the stimulus.[54]

6 Conclusion

This paper explored the macroeconomic effects of financial shocks through a novel lens, highlighting the role of dynamic wage contracts within firms facing credit constraints. By integrating a simple dynamic contracting problem in a general equilibrium model with financial frictions and aggregate shocks, we illustrated how firms adjust wages over time and in response to shocks, depending on their financial conditions. We illustrated the mechanism within a theoretical framework and provided empirical evidence on wage dynamics using matched employer-employee data from Italy.

Our findings show that financially constrained firms tend to backload wages and exhibit higher wage adjustments in response to shocks, thereby enhancing liquidity for investment and job creation. Quantitative analysis revealed that dynamic wage contracts can substantially mitigate the adverse effects of financial shocks on aggregate output, employment, and allocative efficiency. Quantitatively, our model replicates the observed cross-sectional heterogeneity in wage dynamics, and it is at the same time consistent with evidence that the average wage is relatively stable over the cycle. We highlight the policy implications of our findings, particularly in terms of the effectiveness of payroll and investment subsidies during financial crises. Our analysis suggests that incorporating

[53]Calculations from House and Shapiro (2008) show that a 50% bonus depreciation implies a 1.6% investment subsidy for investment goods with a recovery period of 10 years given a nominal interest rate of 3%.

[54]This mechanism is consistent with evidence from Garrett, Ohrn, and Surez Serrato (2020) that did not find a positive effect of investment subsidies on earnings per worker, while documenting positive effects on employment.

the dynamic structure of wage contracts dampens the effects of payroll subsidies while enhancing the effects of investment subsidies.

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Appendix

A Model Appendix and Proofs

A.1 Proof of Proposition 1

First, we show that the measure of workers who search for a job is bounded from above, and that this bound does not depend on the measure of workers M. Then, we will take M such that there must be a positive measure of workers who do not search. Once we show that there is positive measure of workers who do not search, the other results will follow.

The problem of a unmatched entrepreneur before matching and separation, defined in (66), can be written as

$$\widehat{V}(m,z,S) = \max\left(\max_{\theta,W}\left\{[\lambda_f(\theta)J(m,W,z,S) + (1-\lambda_f(\theta))V(m,z,S)]\right\}, V(m,z,S)\right) \tag{38}$$

$$\text{s.t.} \qquad \mathcal{W}(S) \leq \lambda_w(\theta)W + [1-\lambda_w(\theta)]\,\mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]$$

where the constraint can be re-arranged as

$$\frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{\lambda_w(\theta)} \leq W - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right] \tag{39}$$

The optimality conditions of this problem imply

$$\theta) \qquad \lambda_f'(\theta)(J(m,W,z,S) - V(m,z,S)) + \nu\frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{\lambda_w^2(\theta)}\lambda_w'(\theta) = 0$$

$$W) \qquad \lambda_f(\theta)J_W'(m,W,z,S) + \nu = 0$$

Combining the two FOCs, one obtains

$$\theta) \qquad J(m,W,z,S) - V(m,z,S) = -J_W'(m,W,z,S)\frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{\lambda_w}\left(\frac{1-\eta}{\eta}\right)$$

where we have used properties of a Cobb-Douglas matching function[55].
This optimality condition and the constraint jointly determine (W,θ) given workers' values. Market tightness in sub-market (W,θ) is implicitly characterized by

[55] If $\lambda_f = B\theta^{-\eta}$, then $\lambda_f' = -B\eta\theta^{-\eta-1} = -\eta B\lambda_f/\theta < 0$, and $\lambda_w = B\theta^{1-\eta}$ so $\lambda_w' = B(1-\eta)\theta^{-\eta} = B(1-\eta)\lambda_w/\theta$

$$\lambda_w(\theta) = \left(\frac{1-\eta}{\eta}\right) \frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{J(m, W, z, S) - V(m, z, S)} \gamma(m, W, z, S)$$

Then, it must be that the total measure of workers who search, denoted by $s(S)$, is

$$s(S) = \int s(\theta, W)$$

$$s(S) = \int \underbrace{\left(\frac{1}{B}\right)^{-\frac{1}{1-\eta}} \left[\left(\frac{1-\eta}{\eta}\right) \frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{J(m, W, z, S) - V(m, z, S)} \gamma(m, W, z, S)\right]^{-\frac{1}{1-\eta}}}_{\theta^{-1}} \mathbb{1}(m, W, z, S) d\Lambda^m(m, z)$$

where B denotes matching efficiency, and $\mathbb{1}(m, W, z, S)$ is an indicator function equal to one if the entrepreneur opens a vacancy and zero otherwise.
Alternatively, one can write $s(S)$ using the participation constraint to obtain

$$s(S) = \int \underbrace{\left(\frac{1}{B}\right)^{-\frac{1}{1-\eta}} \left[\frac{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{W - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}\right]^{-\frac{1}{1-\eta}}}_{\theta^{-1}} \mathbb{1}(m, W, z, S) d\Lambda^m(m, z) \tag{40}$$

First, note that if the ratio

$$\frac{W - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}{\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]} \tag{41}$$

is bounded in all sub-markets, than $s(S)$ must be bounded, as the measure of workers is 1.

We now show that the ratio (41) is bounded. If there is a positive measure of workers who search, we must have

$$\mathcal{W}(S) - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right] \geq u(\bar{b}) - u(\bar{b}(1-x))$$

so that the denominator of (41) is positive and bounded from below. Note that if there is not a positive measure of workers who search we simply have that $s(S) = 0$, and therefore $s(S)$ is still bounded. Moreover, since the productivity process for productivity follows a discrete Markov process with upper bound $\bar{z}$, it must be that there exists $\bar{W}$ such that no entrepreneurs would ever find it profitable to open a vacancy with $W > \bar{W}$. Therefore, the numerator of (41) is bounded from above:

$$W - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right] \leq \bar{W} - \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right] \leq \bar{W} - \frac{u(\bar{b})}{1-\beta}$$

where the last inequality follows from the fact that at least a positive measure of workers

is searching.
As we assumed that the production function is bounded from above by $\bar{y}$, for large values of capital, we have that $\bar{W}$ is bounded from above

$$\bar{W} \leq \frac{u(\bar{y})}{1-\beta}$$

as no entrepreneurs will be willing to offer a promised utility greater than $\bar{W}$. Note that in one could prove that the ergodic distribution of entrepreneurs' net worth is bounded from above, then W would be bounded without assuming that the production function is bounded.

As a result, the ratio in equation (41) is bounded, as the numerator is bounded from above and the denominator is bounded from below, and both must be positive. Note that the measure of employed workers is bounded above by one, that is the measure of entrepreneurs. Therefore, the measure of workers who search s is bounded as

$$s \leq \underbrace{\left(\frac{1}{B}\right)^{-\frac{1}{1-\eta}} \left[\frac{u(\bar{y})}{(1-\beta)h}\right]^{-\frac{1}{1-\eta}}}$$

which is a function of primitives $\bar{y}, \beta, h, B$. Then there exists a finite measure of workers M that satisfies

$$M > 1 + \underbrace{\left(\frac{1}{B}\right)^{-\frac{1}{1-\eta}} \left[\frac{u(\bar{y})}{(1-\beta)h}\right]^{-\frac{1}{1-\eta}}}_{s}$$

such that a positive measure of workers is not searching. If a positive measure of workers is not searching, this means that in equilibrium workers must be indifferent between searching and not searching. This implies

$$\mathcal{U}(S) = u(\bar{b}) + \beta\mathbb{E}\left[\mathcal{U}\left(S'\right) \mid \mathcal{S}\right]$$

from which it follows that the value $\mathcal{U}$ does not depend on S and it solves

$$\mathcal{U} = \frac{u(\bar{b})}{1-\beta} \tag{42}$$

Moreover, workers being indifferent between searching and not searching also implies

$$u(\bar{b}) + \beta\mathbb{E}\left[\mathcal{U}\left(S'\right) \mid \mathcal{S}\right] = u(\bar{b}(1-x)) + \beta\mathcal{W}(S)$$

and by combining it with equation (42) we get

$$\mathcal{W} = \mathcal{U} + \frac{u(\bar{b}) - u(\bar{b}(1-x))}{\beta}$$

Finally, note that the constraint in (48) does not depend on S anymore, and it simplifies to

$$\frac{u(\bar{b}) - u(\bar{b}(1-x))}{\lambda_w(\theta)\beta} \leq W - \mathcal{U}$$

A.2 Proof of Proposition 2

We guess that the consumption function takes the form

$$c_t = (1-x)m_t \tag{43}$$

with

$$x = \frac{\beta + \beta\gamma}{1 + \beta\gamma}$$

Combining (43) with the risk-sharing condition we obtain

$$w_t = \gamma(1-x)m_t \tag{44}$$

Combining (43), (44) with the law of motion of net worth we obtain

$$m_{t+1} = \frac{1}{1+\gamma(1-x)}\left[k_{t+1}[z_{t+1} + 1 - \delta] - b_{t+1}\right]$$

We consider two cases: the collateral constraint does not bind, or the collateral constraint binds. We will show that in both cases the guess (43) is verified.

- Case 1: the collateral constraint does not bind.
  Here we first guess and verify that the policy function for capital and debt take the form

  $$k_{t+1} = \phi(z_t, \xi_t, q_t)xm_t, \qquad b_{t+1} = -(1 - \phi(z_t, \xi_t, q_t))xm_t$$

  where the function $\phi(z_t, \xi_t, q_t)$ crucially does not depend on net worth m_t.
  Combining this guess with the law of motion of net worth we obtain

  $$m_{t+1} = m_t\frac{x}{1+\gamma(1-x)}\left[\phi(z_t, \xi_t, q_t)[z_{t+1} + 1 - \delta] + (1 + r_t)[1 - \phi(z_t, \xi_t, q_t)]\right] \tag{45}$$

where we denote $1 + r_t = \frac{1}{q_t}$.
We combine (45) with the Euler equation for bonds and capital and obtain

$$1 = \frac{\beta(1+r_t)}{x}\mathbb{E}\left[\frac{1+\gamma(1-x)}{[\phi(z_t,\xi_t,q_t)[z_{t+1}+1-\delta]+(1+r_t)[1-\phi(z_t,\xi_t,q_t)]]}\right]$$
$$1 = \frac{\beta}{x}\mathbb{E}\left[\frac{[1+\gamma(1-x)]\,(z_{t+1}+1-\delta)}{[\phi(z_t,\xi_t,q_t)[z_{t+1}+1-\delta]+(1+r_t)[1-\phi(z_t,\xi_t,q_t)]]}\right]$$

Combining the two Euler equation we are left with

$$\mathbb{E}\left[\frac{z_{t+1}-\delta-r_t}{\phi(z_t,\xi_t,q_t)[z_{t+1}-\delta-r_t]+1+r_t}\right] = 0$$

that provides an equation that implicitly characterizes $\phi(z_t, \xi_t, q_t)$ and also verifies the guess for the functional form of k_{t+1}, b_{t+1}.
Then, take a weighted average of the Euler equations for bonds and capital, with weights $\phi(z_t, \xi_t, q_t)$ and $1 - \phi(z_t, \xi_t, q_t)$, to obtain, after some manipulation:

$$1 = \beta\mathbb{E}\left[\frac{1+\gamma(1-x)}{x}\right]$$

which implies an equation for x:

$$x = \frac{\beta+\beta\gamma}{1+\beta\gamma}$$

that does not depend on net worth.
Next we move to the case when the collateral constraint is binding, and we show that we obtain the same equation for x, that verifies our initial guess (43).

- Case 2: the collateral constraint binds.
  In this case, the law of motion for net worth is

$$m_{t+1} = m_t\frac{1}{1-q_t\xi_t}\frac{x}{1+\gamma(1-x)}\left[z_{t+1}+1-\delta-\xi_t\right] \tag{46}$$

Combining the Euler equation for bonds and capital to substitute for the multiplier on the collateral constraint we obtain

$$v'(c_t^e)[1-q_t\xi_t] = \beta\mathbb{E}\left[v'(c_{t+1}^e)[z_{t+1}+1-\delta\xi]\right] \tag{47}$$

Combining (46) with (47) we obtain:

$$1 = \beta\mathbb{E}\left[\frac{1+\gamma(1-x)}{x}\right]$$

which implies an equation for x:

$$x = \frac{\beta + \beta\gamma}{1 + \beta\gamma}$$

which verifies the initial guess from (43)

A.3 Proof of Proposition 3

Using Proposition 2, entrepreneurs leverage can be expressed as:

$$\frac{b_t}{k_t} = 1 - \frac{1}{\phi(z_t, \xi_t, q_t)}$$

Using Proposition 2 the pass-through of idiosyncratic productivity shocks to wages can be expressed as

$$\frac{\partial\left[\log\left(w_{t+1}\right) - \log\left(w_t\right)\right]}{\partial\left[\log\left(z_{t+1}\right) - \log\left(z_t\right)\right]} = \frac{\phi\left(z_t, \xi_t, q_t\right) z_{t+1}}{\phi\left(z_t, \xi_t, q_t\right)\left[z_{t+1} - \delta\right] + \left(1 - \phi\left(z_t, \xi_t, q_t z_{t+1}\right)\right)\frac{1}{q_t}}$$

from which we find that in the cross-section the pass-through of idiosyncratic shocks to wages is increasing in ϕ. As leverage is increasing in ϕ, then the pass-through is increasing in leverage.

Using Proposition 2 the average growth rate of wages can be expressed as:

$$\mathbb{E}[\log(w_{t+1}) - \log(w_t)] = \mathbb{E}\left[\frac{x}{1 + \gamma(1 - x)}\phi(z_t, \xi_t, q_t)[z_{t+1} + 1 - \delta] + [1 - \phi(z_t, \xi_t, q_t)]\frac{1}{q_t}\right]$$

From which we obtain, after some manipulation, that

$$\frac{\partial\mathbb{E}[\log(w_{t+1}) - \log(w_t)]}{\partial\phi} = \log\left(\frac{1}{\beta}\right) > 0$$

that implies the average growth rate of wages is increasing in entrepreneurs' leverage.

A.4 Proof of Lemma 1

Let denote by γ_0 the multiplier on the constraint defined in (23). The first order conditions of the problem defined in (23) imply:

$$\eta_{s+1}(z^{s+1}, \xi^{s+1}) = \frac{\gamma_0}{v'(c_t^e)} u'(w(z^{s+1}, \xi^{s+1})), \quad \forall s, z^{s+1}, \xi^{s+1}$$

Taking the ratio of the optimality conditions for two different histories we obtain a risk-sharing condition similar to the optimality condition of the recursive problem defined in

(2).

$$\frac{\eta_{s+1}(z^{s+1},\xi^{s+1})}{\eta_{p+1}(z^{p+1},\xi^{p+1})}=\frac{u'(w(z^{s+1},\xi^{s+1}))}{u'(w(z^{p+1},\xi^{p+1}))},\qquad \forall s,p,z^{s+1},\xi^{s+1},z^{p+1},\xi^{p+1}$$

Given the same promised utility W, this implies that the optimal contract that solves (2) is also a solution to the problem defined in (23). To see that, start from $s = t$: as we have $z^{t+1} = z_{t+1}$, $\xi^{t+1} = \xi_{t+1}$ given z_t, ξ_t, the optimal wage contract implied by (2) trivially satisfies the first order conditions of (23). Similarly, one can use the inter-temporal dimension of the risk-sharing condition to check that the optimal wage contract implied by (2) satisfies the optimality condition of (23) for $s > t$.

A.5 Optimal job creation of unmatched entrepreneurs

Using Proposition 1, the problem of a unmatched entrepreneur before matching and separation, defined in (66), can be written as

$$\widehat{V}(m,z,S)=\max\left(\max_{\theta,W}\Big\{[\lambda_f(\theta)J(m,W,z,S)+(1-\lambda_f(\theta))V(m,z,S)]\Big\},V(m,z,S)\right)\tag{48}$$

$$\text{s.t.}\qquad \frac{u(\bar{b})-u(\bar{b}(1-x))}{\beta\lambda_w(\theta)}=W-\mathcal{U}$$

The first order conditions for an interior solution to this problem imply

$$\theta)\qquad \lambda_f'(\theta)(J(m,W,z,S)-V(m,z,S))+\nu\frac{u(\bar{b})-u(\bar{b}(1-x))}{\beta\lambda_w^2(\theta)}\lambda_w'(\theta)=0$$

$$W)\qquad \lambda_f(\theta)J_W'(m,W,z,S)+\nu=0$$

where ν denotes the multiplier on the constraint. Combining the two first order conditions, and replacing the constraint in the first order condition for θ, we obtain:

$$\theta)\qquad J(m,W,z,S)-V(m,z,S)=-J_W'(m,W,z,S)\,[W-\mathcal{U}]\left(\frac{1-\eta}{\eta}\right)$$

where we have used properties of a Cobb-Douglas matching function[56].

[56]If $\lambda_f = B\theta^{-\eta}$, then $\lambda_f' = -B\eta\theta^{-\eta-1} = -\eta B\lambda_f/\theta < 0$, and $\lambda_w = B\theta^{1-\eta}$ so $\lambda_w' = B(1-\eta)\theta^{-\eta} = B(1-\eta)\lambda_w/\theta$

A.6 Walras' Law

If the market for risk-free bonds clears, then the resource constraint holds. First, note that the budget constraint of each matched entrepreneur j can be written as:

$$c_j^e + k_j' \leq y_j + (1-\delta)k_j - w_j - b_j + qb_j'$$

The budget constraint of each unmatched entrepreneur i can be written as:

$$c_i^e \leq -b_i + qb_i'$$

We can integrate the two budget constraints over the measure of matched entrepreneurs j and unmatched entrepreneurs i; and sum the integrated budget constraint to obtain:

$$C^e + K' + W \leq Y + (1-\delta)K - B + qB'$$

where W denotes the sum of all wages paid to workers. Using the market clearing condition for risk-free bonds, that is $B = 0, B' = 0$, and the identity $W = C^w$ as workers are hand-to-mouth, to obtain

$$C^e + K' + C^w \leq Y + (1-\delta)K$$

that is, the resource constraint holds

A.7 Law of motion Γ

The law of motion Γ for the aggregate state S is made of

- An exogenous law of motion for the aggregate shock ξ
- An endogenous law of motion H^m for the distribution $\Lambda^m(m, W, z)$:

$$H^m(\Lambda^m)(\mathcal{M}, \mathcal{W}, \Xi) = \int Q_{\Lambda^m}((m, W, z), \mathcal{M}, \mathcal{W}, \Xi) d\Lambda^m(m, W, z)$$

$$Q_{\Lambda^m}((m, W, z)\mathcal{M}, \mathcal{W}, \Xi) = \sum_{z' \in \Xi} \begin{cases} \pi(z' \mid z) & \text{if } m'(m, W, z; S) \in \mathcal{M}, W'(m, W, z; S) \in \mathcal{W} \\ 0 & \text{otherwise} \end{cases}$$

- An endogenous law of motion H^v for the distribution $\Lambda^v(m, z)$:

$$H^v(\Lambda^v)(\mathcal{M}, \Xi) = \int Q_{\Lambda^v}((m, z), \mathcal{M}, \Xi) d\Lambda^v(m, z)$$

$$Q_{\Lambda^v}((m, z)\mathcal{M}, \Xi) = \sum_{z' \in \Xi} \begin{cases} \pi(z' \mid z) & \text{if } m'(m, z; S) \in \mathcal{M} \\ 0 & \text{otherwise} \end{cases}$$

A.8 Average Job Finding Rate

The average job finding rate in the economy is

$$Average(\lambda_w(\theta)) = \frac{\int \lambda_w(\theta_j) s_j dj}{\int s_j dj}$$

$$Average(\lambda_w(\theta)) = \frac{\int \lambda_w(\theta)\ \theta(m, W, z, S)^{-1} \underbrace{\mathbb{1}(m, W, z, S) d\Lambda^v(m, z)}_{v}}{\int \theta(m, W, z, S)^{-1} \underbrace{\mathbb{1}(m, W, z, S) d\Lambda^v(m, z)}_{v}}$$

The model solution algorithm easily returns $\lambda_f(\theta) = B\theta^{-\eta}$, from which we can get

$$\theta = \left(\frac{\lambda_f(\theta)}{B}\right)^{-\frac{1}{\eta}}, \qquad \lambda_w(\theta) = B\theta^{1-\eta}$$

$$Average(\lambda_w(\theta)) = \frac{\int \lambda_f(\theta) \underbrace{\mathbb{1}(m, W, z, S) d\Lambda^v(m, z)}_{v}}{\int \theta(m, W, z, S)^{-1} \underbrace{\mathbb{1}(m, W, z, S) d\Lambda^v(m, z)}_{v}}$$

B Data Appendix

B.1 Matched employer-employee data

In this section, we provide further details on the matched employer-employee data used for our analysis. We restrict our analysis to firms that participated in the Bank of Italy's annual Survey of Industrial and Service Firms (INVIND). Each year the survey gathers information on investments, gross sales, workforce and other economic variables relating to Italian industrial and service firms with 20 or more employees. More precisely, the survey cover firms operating in the following industries: "Food and beverages", "Textiles and apparel", "Chemical, pharmaceutical, rubber", "Non-metallic minerals", "Metalworking industry", "Wood, paper, furniture", "Water and waste", "Wholesale/retail trade", "Hotels and restaurants", "Transportation and telecommunication", "Other (real estate etc.)".

The National Social Security Institute (*Istituto Nazionale di Predivenza Sociale*, INPS) was asked to provide the complete works histories of all workers that ever transited in an INVIND firm. While the data included spells of employment in which workers were employed at firms not listed in the INVIND survey (e.g., they were employed at an INVIND firm in 2010, but then they changed job in 2012), we restrict our attention to INVIND firms for which we have information on the entire population of employees.

B.1.1 Cleaning procedure and sample construction

We start from the balance sheets data of all firms available in Cerved from 2005 to 2019. These are 1,526,216 firms in total. We restrict our sample to these firms that took part of the INVIND survey between 2005 and 2019 (i.e., that have been surveyed by the Bank of Italy and for which we have access to the entire work histories of all their employees). This step restricts our sample to 9,698 firms.

Then, we perform some cleaning of balance sheet data, as to remove extreme values and clearly implausible entries. We drop firms that have negative entries for value added in at least one year between 2005 and 2019. These are 1,949. We are left with 7,749 firms. We exclude 484 firms with intermittent participation in Cerved and drop 233 firms that appear in the data for less than 3 years (notice that the IV estimation of pass-through coefficients requires at least 3 years of data). We are left with 7032 firms.

For the estimation of pass-through coefficient we run a simple regression to remove persistent heterogeneity across firms, as well as aggregate and sector-specific trends:

$$\log(VA_{it}/L_{it}) = \alpha_i + \text{2digsector-timeFE} + \varepsilon_{it} \tag{49}$$

and get the first difference of residuals. We further exclude 601 firms with residual changes in VA per worker greater than 1 and 661 with changes smaller than -1. This leaves us with 6129 firms. Then, we drop firms for which Cerved declares that debt-related information is not reliable. Cerved classify some firms with non-reliable debt information whenever these firms are not required by the law to report detailed information on their debt in their balance sheets. This step leads us to remove 756 firms. Finally, we also clean from outliers in leverage, as there are firms with values greater than 100 or negative leverage. This leaves us with 5107 firms.

Then, we move to worker-level data. For each worker that has ever transited by an INVIND firm, we have access to their entire work history. We select a 25% random sample of workers from the sample. These are 2,521,206 workers. By merging the balance sheet data and the worker-level data, we find that basically all firms in our sample have at least one worker in the 25% sample. Once we focus on the sub-sample of firms that we obtained from the cleaning procedure we are left with 1,153,746 workers. We exclude 208 workers who have some duplicate record in the dataset. A duplicate record is defined by the combination of earnings $\times$ type of contract $\times$ number of weeks $\times$ which months she worked at the same firm in the same year. This leaves us with 1,153,538 As our empirical analysis is focused on stayers, we only keep workers who in a given year have only one employer, and work 52 weeks. By considering workers who only have one employer in one year, we drop 97,782 workers. This leaves us with 1,055,756 workers. We drop workers who only have part-time contracts during our observation window. This is because this workers may be more affected by an hours response during a value added shock. This leads us to drop 252,719 workers. This leaves us with 803,037 workers.

B.2 Aggregate data

In Section 5 we use time-series data for several macro-economic aggregates. The data come mainly from OECD (variable codes in parenthesis), and we retrieved them from FRED, Federal Reserve Bank of St. Louis.

- Corporate debt: we measure corporate debt as the amount outstanding of total debt securities issued by corporations in the non-financial sector, including all maturities, such that the residence of the issuer is in Italy ('TDSAMRIAONCIT'). Data are available at quarterly frequency.

- Output: we measure aggregate output as real gross domestic product ('CLVMNAC-SCAB1GQIT') at quarterly frequency.

- Unemployment: we measure it as the unemployment rate ('LRUN64TTITQ156S') for people aged 15-64. Data are available at quarterly frequency.

- Employment: we measure the number of employed people aged 15-64 ('LFEM64TTITQ647S'). Data are available at quarterly frequency.

- Hours: we measure hours as the average annual hours worked by employed persons ('AVHWPEITA065NRUG'). Data are available at annual frequency, and we use linear interpolation to construct a measure of hours at quarterly frequency.

- Labor: we measure total labor inputs as the product of employment and hours.

- Investment: we measure investment as grossed fixed capital formation ('ITAGFCFQDSNAQ'). Data are available at quarterly frequency.

B.3 Sample selection: mobility probit

We provide additional details for the sample selection correction that we discussed in Section 4.4. In practice, we estimate a probit regression where on the left-hand side there is a dummy equal to one if the completed tenure $CT_{ij(t_0)}$ of worker i at firm j hired in t_0 is greater or equal than S (i.e., the worker is included in the sub-sample of stayers). The variable $Displaced_{ij(t_0)}$ is equal to 1 if the current job started after a mass layoff or a firm closure at the previous employer. We include in $X_{ij(t_0)}$ the same set of controls that we include in our main specification.

$$Pr(CT_{ij(t_0)} \geq S) = \Phi\Big(\beta_0 + \beta_1 Displaced_{ij(t_0)} + \beta_2 1(lev_{jt_0-1} > \text{median}) + \beta_3 X_{ij(t_0)} + \varepsilon_{ij(t_0)}\Big) \tag{50}$$

We report detailed estimates of equation (50) in Table (9). We discussed the estimates for the coefficient on displaced workers in Section 4.4. Other coefficients also have the expected sign. Workers employed by more productive firms –higher VA/worker– are more likely to stay longer in the firm. Similarly, younger workers, female and managers are more like to have shorter tenure.

To control for sample selection we include the inverse Mills ratio implied by equation (50) as a control in the main regression by interacting it with tenure.

$$\begin{aligned}(\log w_{ij(t_0)t} - \log w_{ij(t_0)t_0}) = &\sum_{s=1}^{S} \tilde{\beta}_s 1(T_{ij(t_0)t} = s) \\ &+ \sum_{s=1}^{S} \tilde{\gamma}_s 1(T_{ij(t_0)t} = s) 1(lev_{jt_0-1} > \text{median}) \\ &+ \sum_{s=1}^{S} \tilde{\delta}_s 1(T_{ij(t_0)t} = s) X'_{ijt_0} \\ &+ \sum_{s=1}^{S} \tilde{\imath}_s 1(T_{ij(t_0)t} = s) \frac{\phi_{ij(t_0)}}{\Phi_{ij(t_0)}} + (u_{ij(t_0)t} - u_{ij(t_0)t_0}) \end{aligned} \tag{51}$$

Variable	Coefficient	Std. Err.
Displaced	-0.1987***	0.0320
$1\left(lev_{jt_0-1} > \text{ median }\right)$	0.0874***	0.0087
log Assets	0.0104***	0.0023
log VA/worker	0.3526***	0.0047
Year of birth		
1960	0.0209	0.0233
1961	-0.0112	0.0234
1962	0.0055	0.0239
1963	-0.0053	0.0250
1964	-0.0468*	0.0244
1965	0.0688**	0.0248
1966	-0.0477*	0.0248
1967	-0.0393	0.0247
1968	-0.0751***	0.0243
1969	-0.0874***	0.0239
1970	-0.1164***	0.0236
1971	-0.1171***	0.0233
1972	-0.2058***	0.0228
1973	-0.2271***	0.0227
1974	-0.2720***	0.0222
1975	-0.3394***	0.0224
1976	-0.3463***	0.0228
1977	-0.4444***	0.0227
1978	-0.4180***	0.0228
1979	-0.4352***	0.0231
1980	-0.4540***	0.0238
Occupation		
Blue Collar	1.1960***	0.0565
High Skilled Blue Collar	1.4174***	0.1019
White Collar	1.2094***	0.0563
Middle Manager	1.2667***	0.0590
Manager	0.2413***	0.0702
Others	0.0413**	0.0921
Female	-1.2361***	0.0748
Sectors FE	Yes	
N	168,184	

Table 9: Estimated coefficients and standard errors for the mobility probit.

B.4 Robustness: wage backloading and interest coverage

Most of the empirical analysis presented in Section 4 focuses on heterogeneity in wage dynamics across firms with different leverage ratios. This approach is motivated both by our model presented in Section 2, where firms borrowing capacity is restricted by the collateral value of their capital, and by evidence that bank credit is largely collateralized in the Italian market (Garrido, Kopp, and Weber, 2016).

In practice, there might be other mechanisms that make firms more or less financially constrained independently of the collateral value of their assets. A common type of debt covenants –provisions in debt contracts that constrain future lending – is related to interest coverage (Greenwald, 2019). In practice these covenants impose a cap on the ratio of a firms interest payments to its earnings or EBITDA.

We show that the empirical results presented in Section 4.4 are robust to classifying firms as more or less financially constrained based on the ratio of their interests payments to their earnings. We propose a measure of interest coverage ("IC") consistent with the structure of these debt covenants, that is defined as the ratio of firms' financial expenses to EBITDA. Our measure of financial expenses is the sum of: interest paid on bank loans, bonds, and other types of financing, interest paid on overdraft accounts, costs of issuing debt instruments, bank fees and service charges related to loans.

$$
\begin{aligned}
(\log w_{ij(t_0)t} - \log w_{ij(t_0)t_0}) = &\sum_{s=1}^{S} \tilde{\beta}_s 1(T_{ij(t_0)t} = s) \\
&+ \sum_{s=1}^{S} \tilde{\gamma}_s 1(T_{ij(t_0)t} = s) 1(IC_{jt_0-1} > \text{median}) \\
&+ \sum_{s=1}^{S} \tilde{\delta}_s 1(T_{ij(t_0)t} = s) X'_{ijt_0} + (u_{ij(t_0)t} - u_{ij(t_0)t_0}) \qquad (52)
\end{aligned}
$$

We estimate equation (52), that is we modify equation (25) from Section 4.4 in order to estimate different wage-tenure profiles for workers employed by firms whose IC ratio at $t_0 - 1$ is above or below the median. We report estimates in Table 10. Estimates for the mobility probit are reported in Column 1. Estimates for the coefficients $\{\tilde{\gamma}_s\}$ are reported in Columns 2 –without controlling for endogenous mobility– and 3 –controlling for endogenous mobility.

The estimated coefficients are qualitatively similar to those presented in Section 4.6: wages grow faster over time at more financially constrained firms. The magnitudes of the estimated coefficients are also similar to those reported in Table 2 for the baseline specification over the first two years of tenure.

Wage growth and interest coverage			
	Probit	Cumulative wage growth	
	(1)	(2)	(3)
$1\left(IC_{jt_0-1} > \text{ median }\right)$	0.0364^{***}		
	(0.0123)		
$\text{Tenure} = 1 : 1\left(IC_{jt_0-1} > \text{ median }\right)$		0.0184^{***}	0.0158^{***}
		(0.0019)	(0.0018)
$\text{Tenure} = 2 : 1\left(IC_{jt_0-1} > \text{ median }\right)$		0.0261^{***}	0.0221^{***}
		(0.0023)	(0.0022)
$\text{Tenure} = 3 : 1\left(IC_{jt_0-1} > \text{ median }\right)$		0.0296^{***}	0.0240^{***}
		(0.0024)	(0.0023)
$\text{Tenure} = 4 : 1\left(IC_{jt_0-1} > \text{ median }\right)$		0.0258^{***}	0.0186^{***}
		(0.0026)	(0.0025)
Displaced	-0.1550^{***}		
	(.0676)		
Tenure=1: Inverse Mills ratio			-0.0047
			(0.0096)
Tenure=2: Inverse Mills ratio			0.0051
			(0.0116)
Tenure=3: Inverse Mills ratio			0.0185
			(0.0121)
Tenure=4: Inverse Mills ratio			0.0165
			(0.0129)
Workers' controls	Yes	Yes	Yes
Firms' controls	Yes	Yes	Yes
N. of workers	130,775	100,680	100,680
N	130,775	402,720	402,720

Table 10: Column (1) reports the results of the probit estimates of whether completed tenure of each worker-firm match is above or below S, using the entire sample of workers hired at t_0. Columns (2) and (3) report the estimates of equation (52) using the sample of workers who stayed at least 4 years. Robust standard errors in parenthesis. The superscripts ***,**, and * denote statistical significance at the 1, 5, and 10 percent level, respectively.

B.5 Robustness: wage backloading and overtime hours

One might be concerned that the estimates of equation (25) presented in Section 4.4 could be influenced by adjustments in hours worked. Indeed, we only observe average monthly earnings and not the base wage. Even if we focus on full-time workers employed year-round in all the empirical exercises, firms may adjust over-time hours for these workers. We exploit the nature of our data that includes information from the INVIND survey on average over-time hours at the firm level. For each firm in the sample we have information on the ratio between over-time hours and total hours in any given year.

We estimate a firm-level regression according to equation (53) similar to the main specification for wage growth in equation (25). On the left-hand side H_{jt} is the share of over-time hours at firm j in year t. On the left-hand side the coefficients $\{\tilde{\gamma}_s\}$ measures the difference in the share of over-time hours in year $t_0 + s$ between firms with high leverage in $t_0 - 1$ and firms with low leverage in $t_0 - 1$. We control firms' size and firms' value added per worker by including them in the vector of controls X_{jt_0}, thus using the same firm-level controls that we used to estimate equation (25).

$$\begin{aligned}
H_{jt} = & \sum_{s=1}^{S} \tilde{\beta}_s 1\left(t=s\right) + \sum_{s=1}^{S} \tilde{\gamma}_s 1\left(t=s\right) 1\left(lev_{jt_0-1} > \text{ median }\right) \\
& + \sum_{s=1}^{S} \tilde{\delta}_s 1\left(t=s\right) X_{jt_0} + \left(u_{ij(t_0)t} - u_{ij(t_0)t_0}\right)
\end{aligned} \tag{53}$$

We test wether there are any significant differences in the share of over-time hours between firms with different leverage starting from $t_0 = 2009$. We report point estimates and confidence intervals for the coefficients $\{\tilde{\gamma}_s\}$ in Figure 3. We find that differences in over-time hours over time are small and not statistically significant. To interpret the magnitudes, note that a coefficient of 0.2 means that on average firms with leverage above median have a share of over-time hours 0.2% larger than firms with leverage below the median. Moreover, while differences in wages increase over time we find that differences in over-time hours don't seem to increase over time. To corroborate these findings, in Figure 4 we report the estimates of $\{\tilde{\gamma}_s\}$ using debt-to-output ratio rather than leverage to construct the indicator function on the right-hand side of equation (53). Also in this case we do not find substantial differences in the share of over-time hours between firms with debt-to-output ratio above and below median.

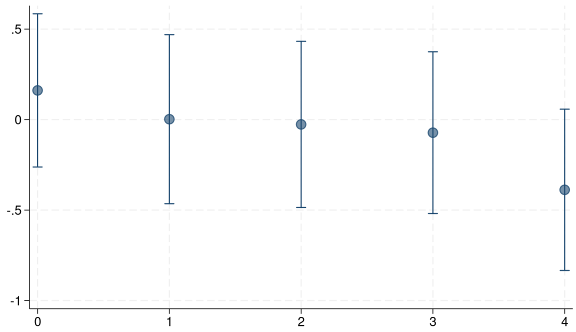


Figure 3: We report estimates of the coefficients $\{\tilde{\gamma}_s\}$ from equation (53) and the 95% confidence intervals.

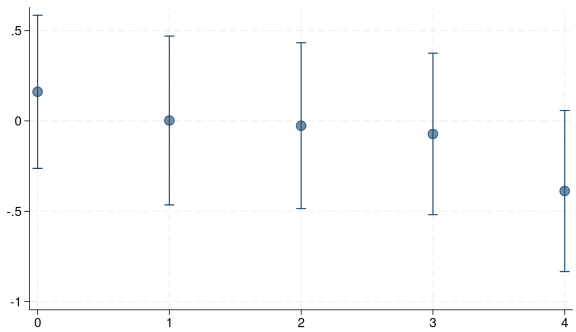


Figure 4: We report estimates of the coefficients $\{\tilde{\gamma}_s\}$ from equation (53) and the 95% confidence intervals using the debt-to-output ratio rather than the leverage ratio to construct the indicator function on the right-hand side of equation (53).

B.6 Robustness: indexes of local credit supply

The province-time index of credit supply follows the methodology proposed in Greenstone, Mas, and Nguyen (2020), using the same data and procedure as in Barone, de Blasio, and Mocetti (2018). Provinces are the natural geographical unit for this exercise, as According to the Italian Antitrust authority the "relevant market" in banking for antitrust purposes is the province (Guiso, Pistaferri, and Schivardi, 2012).

Barone, de Blasio, and Mocetti (2018) use confidential data drawn from the Bank of Italy Supervisory Report database. The data provides information on total outstanding loans –including credit lines, credit receivable and fixed-term loans– extended by Italian banks to the private sector (firms and households) in each province. The data allows us to distinguish between outstanding loans extended to firms or households. Since our aim is to identify more financially constrained firms, we focus on indicators based on firms' loans.

Specifically, for each province p Barone, de Blasio, and Mocetti (2018) estimate the following equation that separates the contribution of bank-specific fluctuations and province-specific fluctuations to bank lending.

$$\Delta \ln (L_{bkt}) = \alpha + \delta_{bt} + \gamma_{kt} + \varepsilon_{bkt}$$

On the left-hand side, the outcome variable is the percentage change in outstanding loans by bank b in province p between the years t and $t-1$. On the right-hand side, γ_{kt} is a set of province-year fixed effects that aim to capture the variation in the change of lending due to purely local economic factors that are common to all banks. The coefficients δ_{bt} capture bank-year fixed effects and represent our parameters of interest, since they capture nationwide bank lending policies that are purged of province-specific loan demand (and of any other province-year level idiosyncratic shock).

$$S_{pt} = \sum_{b} W_{bp}\widehat{\delta}_{bt} \tag{54}$$

We denote by S_{pt} our measure of credit supply in province p in year t. This measure is the weighted average of bank-b-specific changes in outstanding loans δ_{bt} between year t and $t-1$, with the market share W_{bp} of each bank b in province p used as weights. We use different indexes of credit supply using market shares of bank b in province p for different pre-crises years, namely 1999, 2002, and 2006.

We estimate equation (55), that is we modify equation (25) from Section 4.4 to estimate different wage-tenure profiles for workers employed by firms operating in provinces that experienced a more pronounced drop in credit supply. In this sense, we identify as more financially constrained firm j that operates in province p is the index of credit supply at $t_0 - 1$ is below the average of that same year.

$$\begin{aligned}
(\log w_{ijp(t_0)t} - \log w_{ijp(t_0)t_0}) = & \sum_{s=1}^{S} \tilde{\beta}_s 1(T_{ijp(t_0)t} = s) \\
& + \sum_{s=1}^{S} \tilde{\gamma}_s 1(T_{ijp(t_0)t} = s) 1(S_{pt_0-1} < \text{mean}) \\
& + \sum_{s=1}^{S} \tilde{\delta}_s 1(T_{ijp(t_0)t} = s) X'_{ijpt_0} + (u_{ijp(t_0)t} - u_{ijp(t_0)t_0})
\end{aligned} \tag{55}$$

We report estimates of the mobility probit and equation (55) in Table 11, using banks' market shares in 2006 to construct the weights W_{bp}. Note that the estimated coefficients $\{\tilde{\gamma}_s\}$ reported in Column 3 are the same we reported in Table 3.

One concern associated with this approach has to due with large provinces having an effect on nationwide lending policies of certain banks. For instance, this may occur

Wage growth and local credit supply shocks			
	Probit	Cumulative wage growth	
	(1)	(2)	(3)
$1\left(S_{pt_0-1} < \text{mean}\right)$	-0.0842^{***}		
	(0.0141)		
Tenure=1: $1\left(S_{pt_0-1} < \text{mean}\right)$		0.0247^{***}	0.0205^{***}
		(0.0021)	(0.0022)
Tenure=2: $1\left(S_{pt_0-1} < \text{mean}\right)$		0.0384^{***}	0.0346^{***}
		(0.0025)	(0.0027)
Tenure=3: $1\left(S_{pt_0-1} < \text{mean}\right)$		0.0337^{***}	0.0303^{***}
		(0.0027)	(0.0028)
Tenure=4: $1\left(S_{pt_0-1} < \text{mean}\right)$		0.0312^{***}	0.0310^{***}
		(0.0029)	(0.0030)
Displaced	-0.1435^{***}		
	(.0376)		
Tenure=1: Inverse Mills ratio			0.0975^{***}
			(0.0162)
Tenure=2: Inverse Mills ratio			0.0883^{***}
			(0.0191)
Tenure=3: Inverse Mills ratio			0.0753^{**}
			(0.0201)
Tenure=4: Inverse Mills ratio			0.0179
			(0.0217)
Workers' controls	Yes	Yes	Yes
Firms' controls	Yes	Yes	Yes
N. of workers	127,229	98,750	98,750
N	127,229	395,000	395,000

Table 11: Column (1) reports the results of the probit estimates of whether completed tenure of each worker-firm match is above or below S, using the entire sample of workers hired at t_0. Columns (2) and (3) report the estimates of equation (55) using the sample of workers who stayed at least 4 years. Robust standard errors in parenthesis. The superscripts ***,**, and * denote statistical significance at the 1, 5, and 10 percent level, respectively.

Robustness: wage growth and local credit supply shocks				
	Including province p		Excluding province p	
	1999	2002	1999	2002
	(1)	(2)	(3)	(4)
Tenure=1: $1\left(S_{pt_0-1} < \text{mean}\right)$	0.0175***	0.0204***	0.0161***	0.0141***
	(0.0022)	(0.0021)	(0.0021)	(0.0021)
Tenure=2: $1\left(S_{pt_0-1} < \text{mean}\right)$	0.0300***	0.0350***	0.0290***	0.0287***
	(0.0027)	(0.0026)	(0.0025)	(0.0026)
Tenure=3: $1\left(S_{pt_0-1} < \text{mean}\right)$	0.0285***	0.0302***	0.0234***	0.0199***
	(0.0028)	(0.0027)	(0.0027)	(0.0027)
Tenure=4: $1\left(S_{pt_0-1} < \text{mean}\right)$	0.0279***	0.0301***	0.0254***	0.0203***
	(0.0030)	(0.0029)	(0.0029)	(0.0029)
Inverse Mills ratio	Yes	Yes	Yes	Yes
Workers' controls	Yes	Yes	Yes	Yes
Firms' controls	Yes	Yes	Yes	Yes
N	395,000	395,000	395,000	395,000

Table 12: Estimates of the coefficients $\tilde{\gamma}_s$ from equation (55) obtained using alternative methods to construct indexes of credit supply S_{pt}. Robust standard errors in parenthesis. The superscripts ***,**, and * denote statistical significance at the 1, 5, and 10 percent level, respectively.

when a province market is sufficiently large with respect to the national credit market of a certain bank (e.g., small banks are typically geographically concentrated in few provinces) and, therefore, it may affect its lending policy. As a result, it would be hard to plausibly isolate changes in credit supply from S_{pt}. Barone, de Blasio, and Mocetti (2018) address this concern by estimating bank-fixed effects δ_{bt} using all provinces but p. In this way, they isolate the component of bank-specific fluctuations in credit that is not driven by province-specific effects, i.e., a *leave-one-out* approach. In Table 12 we report estimates of the coefficients $\{\tilde{\gamma}_s\}$ from equation (55) using alternative methods to construct indexes of credit supply S_{pt}. In Columns 1 and 2 we report estimates obtained using different banks' market shares in different years –1999 and 2002– to construct indexes of local credit supply. In Columns 3 and 4 we repeat the same exercise but using the *leave-one-out* credit index in Barone, de Blasio, and Mocetti (2018). We find that results are robust to these alternative methods to construct indexes of credit supply.

We consider firms that operate in provinces p with an index of credit supply conditions S_{pt} below the median as more financially constrained.

B.7 Robustness: pass-through coefficients

We present several robustness exercises to further corroborate the results presented in Section 4.6 on how the pass-through coefficient of value added per worker varies with firms' financial conditions.

In Section 4.6 we illustrated how firms with higher leverage pass-through more of changes in value added per worker to wages. This approach is motivated both by our model presented in Section 2, where firms borrowing capacity is restricted by the collateral value of their capital, and by evidence that bank credit is largely collateralized in the Italian market (Garrido, Kopp, and Weber, 2016).

Here we consider alternative ways to classify firms as more or less financially constrained. In practice, there might be other mechanisms that make firms more or less financially constrained independently of the collateral value of their assets. For instance, firms' borrowing capacity can be limited by their earnings as opposed to their assets (Drechsel, 2023; Lian and Ma, 2020). Also, a common type of debt covenants –provisions in debt contracts that constrain future lending – is related to interest coverage (Greenwald, 2019). In practice these covenants impose a cap on the ratio of a firms interest payments to its earnings or EBITDA. Therefore, firms' borrowing capacity can be limited by the ratio between interest payments and earnings.

In this Section we illustrate that estimates presented in Section 4.6 are robust to using alternative methods to classify firms as financially constrained. First, we consider an alternative way to construct the leverage ratio b/k, where the denominator includes only financial debt, excluding debt to suppliers. Results are reported in Column 1 of Table 13. Second, we consider firms with a high debt-to-output ratio to be more financially constrained, consistent with a view of earnings-based constraints as opposed to collateral constraints (Drechsel, 2023). Results are reported in Column 2 of Table 13. Third, we consider firms with a interest coverage ratio to be more financially constrained, consistent with a view of interest coverage covenants (Greenwald, 2019). Results are reported in Column 3 of Table 13. Fourth, as we explained in Section 4.4, one might be concerned that leverage at $t_0 - 1$ may be correlated with firms permanent unobservable characteristics that affect wage growth. For instance, some firms may have better access to credit and persistently sustain high leverage without being financially constrained. To address this concern we consider firms with leverage above their own average to be more financially constrained. Results are reported in Column 3 of Table 13.

B.8 Wage backloading and investment

This section provides descriptive evidence supporting the model mechanism by exploiting differences in wage flexibility across firms. We show that firms with more flexibility in wage setting backloaded wages more during the Great Recession while experiencing a lower drop in investment.

The INVIND survey provides information on whether firms in our sample signed any "second level contract". In the Italian labor market, "second-level contracts" (also

Robustness: pass-through coefficients				
	Leverage ratio (1)	Debt-to-output ratio (2)	Interest coverage (3)	Leverage ratio (4)
$\Delta\varepsilon_{jt}\times$ less constrained	0.045***	0.041***	0.049***	0.046***
	(0.001)	(0.001)	(0.001)	(0.001)
$\Delta\varepsilon_{jt}\times$ more constrained	0.068***	0.070***	0.066***	0.118***
	(0.001)	(0.001)	(0.001)	(0.001)
Within firm variation	No	No	No	Yes
N	11,052,040	11,052,040	11,052,040	11,052,040

Table 13: We split the sample and report estimates of pass-through coefficients for firms that we label as constrained and unconstrained. We use alternative methods to classify firms as more or less financially constrained. Robust standard errors in parentheses. The superscripts ***,**, and * denote statistical significance at the 1, 5, and 10 percent level, respectively.

known as decentralized bargaining agreements) refer to collective agreements negotiated at a level below the national sectoral agreements –typically at the company or regional level. These contracts complement the national collective agreements (known as first-level contracts), which set the baseline conditions for wages, hours, and benefits across an entire sector. These contracts enhance wage flexibility by allowing firms to adjust wages based on specific performance or productivity metrics, as well as regional economic conditions. First, we estimate equation (24) separately for firms that did and did not have any "second level contract" in place during 2008. We plot estimates of γ_s in Figure 5 for these two groups of firms. The sample differs from the one used in Section 4.4 as the information on second level contracts is not available for all firms in our sample. Estimates from (24) show that firms with high leverage and a second level contract during 2008 backloaded wages of newly hired workers substantially more than highly levered firms without a second level contract during 2008. In other words, financially constrained firms with greater flexibility in wage setting offered steeper wage-tenure profiles to workers hired during the Great Recession.

Then, we turn to study the investment dynamics of these different groups of firms during the Great Recession. We estimate a triple-difference specification, as described in equation (56), to measure the differential response of investment during the Great Recession for financially constrained firms with and without second level contracts.

$$\begin{aligned}\log(i/k)_{jt} =& \beta_0 + \sum_s \alpha_s year_s 1(lev_{jt_0-1} > \text{median}) + \sum_s \beta_s year_s 1(2ndC_{jt_0}) \\ & \sum_s \pi_s year_s 1(2ndC_{jt_0}))1(lev_{jt_0-1} > \text{median}) + \sum_s \delta_s X_j' + u_{jt} \end{aligned} \tag{56}$$

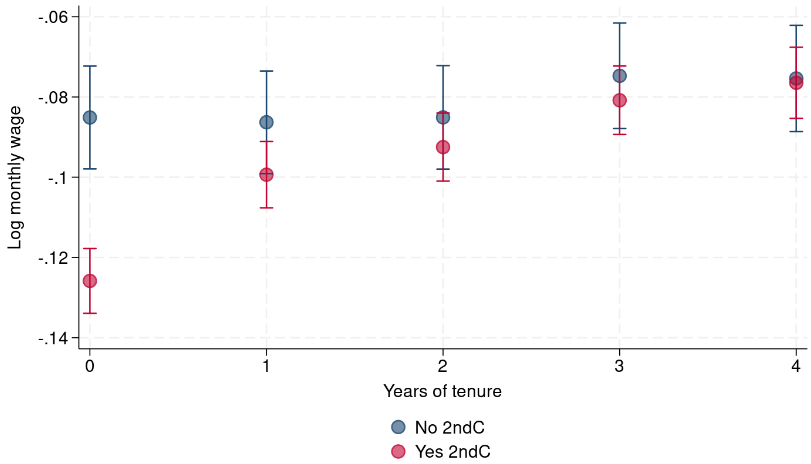


Figure 5: We plot estimates for the coefficients $\{\gamma_s\}_{s=0}^4$ in equation (24), setting $t_0 = 2009$ for firms that had "second level contract" (red) and firms who did not (in blue).

On the left-hand side of (56) there is the logarithm of investment rate of firm j at time t. On the left-hand side, $2ndC_{jt_0}$ is a dummy equal to one if firm j has a second level contract at time t_0. The coefficient π_s measures the difference in $\log(i/k)$ between firms with leverage above median with a second level contract and firms with leverage above median without a second level contract. We control for sector-year fixed effects by including firms' sectors in X_j', as the dynamics of investment rates over time can vary substantially across sectors.

Estimates of the coefficients π_s are plotted in Figure 6. While there is no significative difference between investment rates of firms with and without second level contracts before the Great Recession, estimates of the coefficients π_s during the Great Recession are positive and statistically significant. These estimates imply that that highly levered firms with second level contracts experienced a less pronounced drop in investment during the Great Recession compared to highly levered firms without second level contracts.

This descriptive evidence provides additional empirical support for the model mechanism, as firms with greater flexibility in wage setting backloaded wages more during the Great Recession and experienced a less pronounced drop in investment. These results are consistent with the model mechanism illustrated in Section 3, namely that wage backloading frees resources for investment. In the next section we use our quantitative model to study the macro-economic implications of dynamic wage contracts during financial crises.

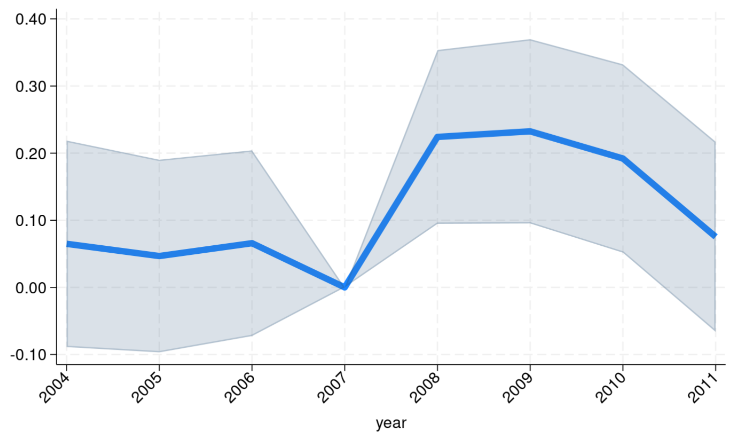


Figure 6: The blue line interpolates estimates for the coefficients $\{\pi_s\}_{s=0}^{4}$ in equation (56), setting $t_0 = 2009$. The gray shaded area corresponds to the 95% confidence intervals.

B.9 Estimation of productivity process

Let consider the following Error Correction Model (ECM)

$$\begin{aligned} y_{jt} &= B_j + z_{jt} + \nu_{jt} \\ z_{jt} &= \rho z_{jt-1} + \varepsilon_{jt} \\ \upsilon \sim (0, \sigma_\upsilon)\,, \quad & \varepsilon \sim (0, \sigma_\varepsilon)\,, \quad B \sim (0, \sigma_B) \end{aligned} \tag{57}$$

where log-productivity y_{jt} is the sum of a firm-specific component B_j, a persistent component z_{jt} and a purely idiosyncratic component ν_{jt}. Using panel data and assumptions to recover firms' productivity from balance sheet data, one can estimate the parameters of the stochastic process described in (57) using a generalized method of moments estimator. Indeed, the parameters are over-identified from auto-covariances of firms' productivity y_{jt} at different horizons, as in standard income processes (Heathcote, Perri, and Violante, 2010).

The goal of this exercise is to recover estimates of ρ and σ_ε, that are the parameters disciplining the stochastic process for idiosyncratic productivity in our model. At the same time, this exercise allows us to isolate variation in productivity driven by a persistent components from cross-sectional variation in firms' productivity driven by fixed heterogeneity and purely idiosyncratic shocks, as these features are not part of our model. Once we have a proxy for firms' log-productivity y_{jt}, the stochastic process specified in equation (57) is identified from panel data and can be estimated using a generalized method of moments estimator.

We use data for a balanced panel of firms between 2007 and 2018. Consistently with the production function in our model we measure firms' productivity as

$$y_{jt} = \log(\text{VA/worker})_{jt} - \alpha \log(k)_{jt}$$

We use fixed assets at book value to measure k_{jt}. We report also estimates obtained using total assets at book value to measure k_{jt}, as well estimates in which we simply measure y_{jt} as the log of value added per worker. Estimates are reported in Table 14. In all these cases we obtain similar estimates. We obtain estimates of the parameters disciplining the stochastic process at quarterly frequencies as:

$$\sigma^2_{\varepsilon,\text{quarter}} = \frac{\sigma^2_{\varepsilon,\text{annual}}}{4}$$
$$\rho_{\text{quarter}} = \rho^{1/4}_{\text{annual}}$$

	(1) Fixed assets	(2) Total assets	(3) VA/worker
ρ	0.86	0.86	0.88
σ_ε	0.16	0.13	0.13
	Quarterly frequencies		
	Fixed assets	Total assets	VA/worker
ρ	0.96	0.96	0.96
σ_ε	0.08	0.07	0.07

Table 14: Estimates of ρ, σ_ε obtained using different specifications. The first panel reports estimates at annual frequency, as balance sheet data. The second panel reports the implied estimates at quarterly frequency.

C Quantitative details and additional results

C.1 Numerical algorithm

We solve the model using standard projection methods and we approximate the law of motion of the aggregate state S following Krusell and Smith (1998). Because of Proposition 1, the problem of entrepreneurs depends on the aggregate state S only through the realization of the aggregate shock ξ and the price of bonds q. While the law of motion for ξ is exogenous, we rely on the following approximation to characterize the law of motion of q as:

$$q_{t+1} = \beta_0(\xi_t, \xi_{t+1}) + \beta_1(\xi_t)q_t \tag{58}$$

that is defined by the six coefficients: $\beta_0(\xi_L, \xi_L), \beta_0(\xi_L, \xi_H), \beta_0(\xi_H, \xi_L), \beta_0(\xi_H, \xi_H), \beta_1(\xi_H), \beta(\xi_L)$. In other words, agents forecast future prices conditional on future realizations of the aggregate shock ξ_{t+1} using information on current prices and the current value of the aggregate shock ξ_t.

In this sense, we summarize the aggregate state in period $t+1$ as (ξ_{t+1}, ξ_t, q_t), where q_t depends on the history of previous shocks ξ according to equation (58). We start the algorithm with an initial guess for the coefficients in (58). As there is not an explicit characterization for q as a function of a one-dimensional aggregate state variable, we follow Krusell and Smith (1997) and we include the price of bonds q as a state variable in the entrepreneurs' decision problem. This means that in a first step we solve for entrepreneurs' decision problem as a function of q, and then we solve for the market clearing price period by period using entrepreneurs' decision rules during simulation.

We simplify the computation by solving the problem of matched entrepreneurs defined in (2) using the multiplier γ rather than the promised utility W as a state variable. The advantage from doing so is that, according to the optimality conditions for $W'(z', \xi')$, the multiplier γ has to be constant throughout the length of a match. Moreover, it's also easier to solve for the optimal policy for state contingent wages $w'(z', \xi')$ using the risk-sharing condition given the multiplier γ.

This way, one can solve for the value functions without explicitly characterizing the state-contingent future promised values $W'(z', \xi')$. More formally, one can define the Pareto problem $P(m, \gamma, z, S)$ as

$$P(m, \gamma, z) = \max_W \Big[J(m, W, z) + \gamma W \Big]$$

and one can easily show that the policy functions that solve the program defined in (59) are a solution to (2).

$$P(m,\gamma,z,S) = \max_{\substack{c^m,b',k',\\ m'(z',\xi'),w(z',\xi')}} \Big\{ v(c^m) + \beta\phi\mathbb{E}\left[V(m'(z',\xi'),z',S')|z,S\right] + \gamma\mathbb{E}[u(w'(z',\xi'))] + \beta(1-\phi)\Big\{\mathbb{E}\Big[P(m',\gamma,z',S')|z,S\Big]\Big\}\Big\} \tag{59}$$

$$\begin{aligned}
&(\text{Budget constraint}: \lambda^e) && c^m + k' \leq m + qb' \\
&(\text{Net worth}: \eta(z',\xi')) && m'(z',\xi') \leq z'f(k') + (1-\delta)k' - w'(z',\xi') - b' \\
&(\text{Collateral constraint}: \mu) && b' \leq \xi k'
\end{aligned}$$

We solve the problem of matched entrepreneurs using grids for the state variables (m,γ,z,ξ,q), and the problem of unmatched entrepreneurs with using grids for the states (m,z,ξ,q). We use a GPU to iterate over policy functions and value functions, given the relatively large number of states. Similarly to Menzio and Shi (2011), we impose $\lambda_f(\theta) = \min(1, B\theta^{-\eta})$. Note that the participation constraint of workers searching in sub-market (θ, W) always implies $\lambda_w(\theta) \in (0,1)$[57]. We initialize the algorithm with an initial guess $W(m,\gamma,z,\xi,q)$ for the promised utility as a function of the states, and we update this guess at each iteration using the wage policy and the previous guess of W.

Then, at each step we combine the value function of matched entrepreneurs obtained as a function of (m,γ,z,ξ,q) with the corresponding promised utility $W(m,\gamma,z,\xi,q)$ to obtain policy functions and value functions that depends on the states (m,W,z,ξ,q). We then use these as inputs to update the value functions and the policy functions of unmatched entrepreneurs.

Once we have solved for the policy functions and the value functions, we simulate the model by approximating the distribution of idiosyncratic states $\Lambda^m(m,W,z), \Lambda^v(m,z)$ on a grid. We solve for the market clearing price of risk-free bonds q period-by-period during simulation, that is we solve for q such that

$$\sum_{m,W,z} b'(m,W,z,\xi,q) \times \Lambda^m(m,W,z) = \sum_{m,z} a'(m,z,\xi,q) \times \Lambda^v(m,z)$$

We simulate the economy for T periods, we drop the first T_0 observations, and we use the simulated series of prices q to update the coefficients of the forecasting rule (58). We keep iterating until the root mean squared error obtained from using the initial guess for

[57] We calibrated and solved an alternative version of our model using the matching function proposed by den Haan, Ramey, and Watson (2000), that implies job finding rates and vacancy filling rates always below one. We find very similar results.

(58) to predict q is small enough, that is when the agents forecasting rule is accurate and consistent with the time series of prices. We stop the algorithm when the R^2 from the forecasting regression (58) on newly simulated data is greater than 0.999.

When we solve the model with payroll subsidies we also include government debt as an aggregate state variable in agents' decision problem. Entrepreneurs also need to forecast future values of public debt. Assuming that government debt follows an AR(1) with persistency ρ_B greatly simplifies the forecasting problem.

C.2 Model with subsidies: details

The problem of matched entrepreneurs in the economy with payroll and investment subsidies have been described in (33). Compared to the problem of matched entrepreneur in the baseline model, there are two additional idiosyncratic sate variables (k, e). The state variable e is equal to one if the entrepreneur is eligible for the subsidies, and zero otherwise. Its law of motion is given by

$$\begin{aligned} e' &= e \quad \text{if } \xi' = \xi_L \\ e' &= 0 \quad \text{if } \xi' = \xi_H \end{aligned}$$

meaning that eligible entrepreneurs remain eligible during recessions, and all matched entrepreneurs become ineligible at the end of a recession.

Unmatched entrepreneurs face a discrete choice problem between posting a vacancy or not. Entrepreneurs that decide to open a vacancy have to choose a sub-market (θ, W) where to open it. Their problem is described in (60). If a unmatched entrepreneur is matched to a worker, he obtains the continuation value $J(m, W, z, 0, 1, S)$, that is the value of being a matched entrepreneur with 0 capital and eligible for subsidies $(e = 1)$.

$$\widehat{V}(m, z, S) = \max\left(\max_{(\theta,W)}\left\{ [\lambda_f(\theta)J(m, W, z, S) + (1-\lambda_f(\theta))V(m, z, S)] \right\}, V(m, z, S)\right) \tag{60}$$

After matching and separation, unmatched entrepreneurs decide how much to consume and how much to save, according to (61). The main difference compared to the model with no subsidies is that lump sum transfers enter the law of motion of net worth.

$$V(m,z,S) = \max_{a',c^e,m'} \left\{ v\left(c^e\right) + \beta\mathbb{E}\left[\widehat{V}\left(m',z',S'\right) \mid z,S\right] \right\} \tag{61}$$

$$\text{(Budget constraint)}: \quad c^e + qa' \leq m$$

$$\text{(Net worth)}: \quad m' \leq a' + \bar{b} + T'$$

The decision problem of workers is unchanged from the baseline model described in Section 2.

The market clearing condition for risk-free bonds is now:

$$\int b'(m,W,z,k,e,S)d\Lambda^m(m,W,z,k,e) + B' = \int a'(m,z,S)d\Lambda^v(m,z)$$

Government expenditure G on payroll subsidies and investment subsidies is equal to

$$G_t = \int \mathbb{E}\left[w\left(m_{t-1},W_{t-1},z_{t-1},k_{t-1},e_{t-1},S_{t-1},z_t,S_t\right) \mid z_{t-1}\right] \times \tau_N\left(\xi_t\right) d\Lambda^m_{t-1}\left(m_{t-1},W_{t-1},z_{t-1},k_{t-1},1\right)$$
$$+ \int i\left(m_t,W_t,z_t,k_t,e_t,S_t\right) \times \tau_I\left(\xi_t\right) d\Lambda^m_t\left(m_t,W_t,z_t,k_t,1\right)$$

C.3 Additional counterfactual: changing workers' risk aversion

Wage contracts in this model solve a risk-sharing problem between entrepreneurs and workers. Section 3 highlighted that the magnitude of wage adjustments depends on the ratio between the risk-aversion coefficient of workers and entrepreneurs σ_W/σ_E. When this ratio is high, wage adjustments must be low, and entrepreneurs have to bear more risk. The structure of wage contract affects entrepreneurs' investment and hiring decisions, as illustrated in Section 3, and the magnitudes are quantitatively relevant, as shown in Section 5. Here we use the quantitative model to further illustrate the role played by wage adjustments in shaping the dynamics of investment, employment, and output after an aggregate financial shock. We compare the calibrated model with an economy where workers have a higher coefficient of relative risk aversion. Figure 7 plots the impulse response functions to a drop in ξ in the baseline model ($\sigma_W = 11$, solid line) and in a model with a higher value for the relative risk aversion coefficient of the workers ($\sigma_W = 22$, dashed line). We recalibrated the value of ξ_L in the counterfactual economy with higher σ_W as to obtain the same drop in aggregate debt. Although the two economies experience the same drop in aggregate debt, output falls more in the economy with higher σ_W, that is when firms adjust wages less. The results are qualitatively similar to those presented in Figure 2 from Section 5: output decreases more in response to an aggregate financial shock when we constrain firms' ability to backload wage payments and to adjust wages in response to shocks.

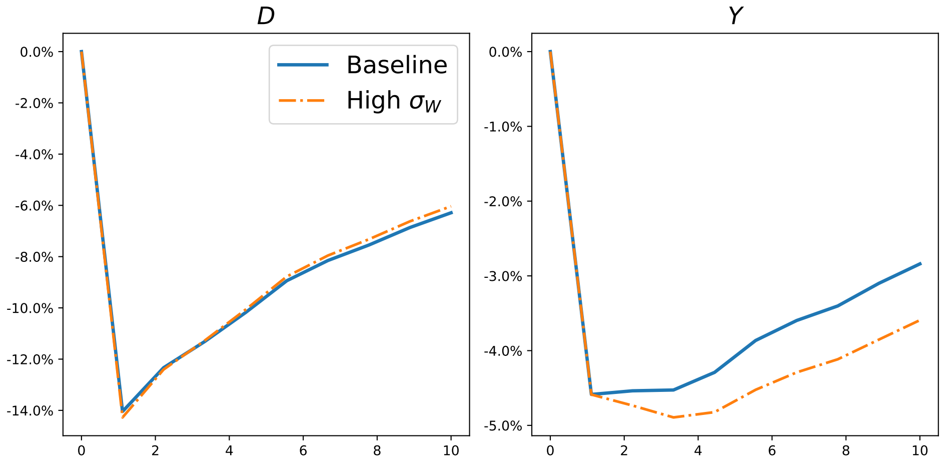


Figure 7: We compute $2 \times M$ simulations of length T. We draw M sequences of uniform random numbers that we use to simulate realizations of ξ. In the first M simulations we set $\xi = \xi_L$ at $T - 10$. The IRFs are computed taking the difference in logs between the first and second set of simulations from $T - 10$ to T, averaging across M.

C.4 Additional counterfactual: Nash bargaining

In this section we compare the response to an aggregate financial shock between the baseline model presented in Section 2 and a counterfactual economy where wages are re-contracted period by period with Nash bargaining.

In Section 5 we presented one of the main quantitative results of the paper, namely that firms' ability to adjust the cost of labor substantially reduces the output cost of a credit crunch. To this end, we compared the baseline model with a counterfactual economy in which firms cannot commit to future wages. This counterfactual economy is ideal for quantitatively evaluating the mechanism presented in Section 3. Indeed, the counterfactual model has two features of a standard spot labor market: firms cannot choose to adjust wages over time and in response to shock, and the allocative wage for job creation is the wage at the time of hiring. This section serves a different purpose, as it aims to quantify how much our economy with optimal dynamic wage contracts differs from other common wage-setting protocols within long-term employment relationships. To this end, we consider a counterfactual economy where wages are re-contracted period by period with Nash bargaining.

Surplus sharing rule

Consider an economy in which, at the end of each period, the entrepreneur and the worker negotiate a wage for the subsequent period with efficient bargaining weights as in Hosios (1990). The outcome of this simple bargaining problem leads to the common surplus sharing rule in equation (62). Note that in this setting, the value of being a matched entrepreneur depends only on the state variables (m, z, S) since wages are recontracted period by period. Moreover, both the value of an employed worker $\mathcal{W}$ and wages are a function of the state variables of the entrepreneur (m, z, S).

$$\frac{J(m,z,S) - V(m,z,S)}{W(m,z,S) - \mathcal{U}} = \frac{\mathbb{E}[\eta(m,z,S)(z',\xi')|z,S]}{u'(w'(m,z,S))}\left(\frac{1-\eta}{\eta}\right) \tag{62}$$

The value of an employed worker $W(m, z, S)$ is defined recursively given the policy function of matched entrepreneurs $m'(m, z, S)$ as:

$$W(m,z,S) = u(w(m,z,S)) + \beta\mathbb{E}\Big[(1-\phi)W'(m'(m,z',S'),z',S') + \phi\mathcal{U}|z,S\Big] \tag{63}$$

Matched entrepreneurs

Matched entrepreneurs solve their problem given the wage schedule that solves (62). In other words, they choose consumption, capital, and bonds to maximize the present discounted value of their utility.

$$\begin{aligned} J(m,z,S) = \max_{c^e,b',k',m'(z',\xi')} \Bigg\{ & v(c^e) + \beta(1-\phi)\underbrace{\mathbb{E}\left[J(m'(z',\xi'),z',S')|z,S\right]}_{\text{not separate}} \\ & + \beta\phi\underbrace{\mathbb{E}\left[V(m'(z',\xi'),z',S')|z,S\right]}_{\text{separate}}\Bigg\} \end{aligned} \tag{64}$$

$$\begin{aligned} &(\text{Budget constraint} : \lambda^e) && c^e + k' \leq m + qb' \\ &(\text{Net worth} : \eta(z',\xi')) && m'(z',\xi') \leq z'f(k') + (1-\delta)k' - w'(m,z,S) - b' \\ &(\text{Collateral constraint} : \mu) && b' \leq \xi k' \end{aligned}$$

Search

In the labor market, there is directed search, and workers have full information on the entrepreneurs' state variables. In other words, each worker who searches for a job decides to search for an entrepreneur with state variables (m, z). A non employed worker who looks for a job with a firm that has state variables (m, z) will find a job with probability $\lambda_w(\theta(m, z, S))$ and the associated value $W(m, z, S)$ is implied by the surplus sharing rule (62) and equation (63). In equilibrium, a worker will search for a job with an entrepreneur indexed by (m, z) if and only if it is weakly better than searching for a job with any other entrepreneur. This implies that as in the baseline model, one can represent the labor market with a continuum of sub-markets indexed by (θ, W). Therefore, the problem of a worker who searches implies that workers search in a given sub-market $(\tilde{\theta}, \tilde{W})$ if and only if it is weakly better than searching in any other sub-market, that is:

$$\underbrace{\lambda_w(\tilde{\theta})\tilde{W} + \left[1 - \lambda_w(\tilde{\theta})\right] \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]}_{\mathcal{W}} \geq \max_{(\theta, W)} \left\{\lambda_w(\theta)W + \left[1 - \lambda_w(\theta)\right] \mathbb{E}\left[\mathcal{U}\left(S'\right) \mid S\right]\right\} \quad (65)$$

Note that given $\mathcal{W}, W(m, z, S)$, the probability of being matched $\lambda_w(\theta(m, z, S))$ is determined according to (65) for any (m, z, S). If equation (65) is a strict inequality, then $\lambda_w(\theta(m, z, S)) = 0$; if equation (65) holds with equality, then it can be solved for $\lambda_w(\theta(m, z, S))$. As in the baseline economy, the left-hand side of equation (65) can be easily obtained under Proposition 1 which characterizes $\mathcal{U}$ and $\mathcal{W}$ as functions of parameters.

Vacancy posting decision

Before matching and separation, unmatched entrepreneurs may choose to post a vacancy or not. Entrepreneurs who decide to open a vacancy will be matched with a worker with probability $\lambda_f(\theta(m, z, S))$, where the probability depends on the state variables of the entrepreneurs. With the complementary probability, the entrepreneur remains unmatched, where $V(m, z, S)$ denotes as usual the value of being unmatched in the afternoon, after matching and separation.

$$\widehat{V}(m, z, S) = \max\left(\left\{\, [\lambda_f(\theta(m, z, S))J(m, z, S) + (1 - \lambda_f(\theta(m, z, S)))V(m, z, S)]\,\right\}, V(m, z, S)\right) \quad (66)$$

After matching and separation, unmatched entrepreneurs decide how much to consume and how much to save, according to (4), which is the same as in the baseline model.

Quantitative analysis

Figure 8 plots the impulse response functions to a drop in ξ in the baseline model and in the counterfactual economy where wages are recontracted period by period with Nash bargaining. As usual, we recalibrated the value of ξ_L in the counterfactual economy to obtain the same drop in aggregate debt. Although the two economies experience the same drop in aggregate debt, output falls more in the economy with Nash bargaining. The results are qualitatively similar to those presented in Figure 2 from Section 5: output decreases more in response to an aggregate financial shock when we constrain firms' ability to backload wage payments and to adjust wages in response to shocks. Interestingly, in this counterfactual economy firms do adjust wages over time and in response to shocks, as they are renegotiated period by period. However, in the counterfactual economy, firms cannot commit to specific future wage increases beyond what would be optimal according to the surplus sharing rule (62).

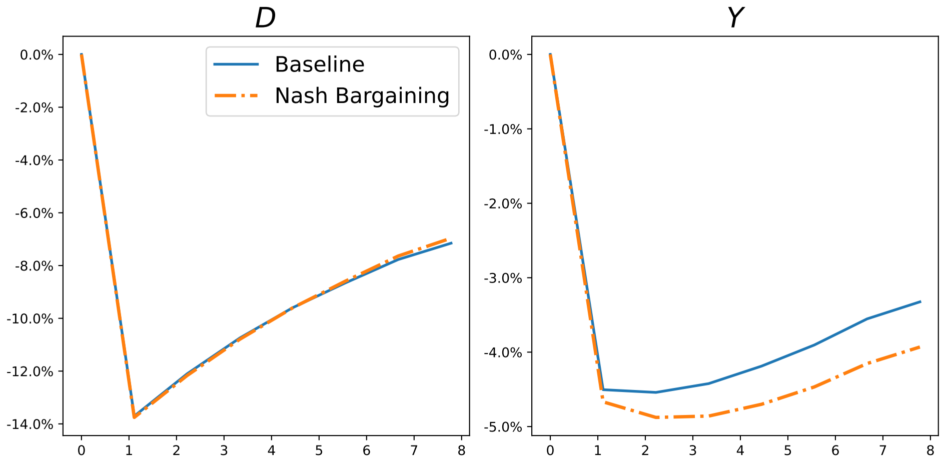


Figure 8: We compute $2 \times M$ simulations of length T. We draw M sequences of uniform random numbers that we use to simulate realizations of ξ. In the first M simulations we set $\xi = \xi_L$ at $T-10$. The IRFs are computed taking the difference in logs between the first and second set of simulations from $T-10$ to T, averaging across M.

C.5 Wage ciclicality

The measurement of wage cyclicality remains a significant area of research, with findings being highly sensitive to the specific definition of ”wage” employed (Basu and House, 2016) and the availability of granular data on worker compensation (Grigsby, Hurst, and Yildirmaz, 2021). Nevertheless, there is broad consensus regarding well established stylized facts.

In this section we emphasize that the quantitative model is consistent with these stylized facts on wage ciclicality. First, in our model the average wage moves little during recession (Grigsby, 2022). Second, the user cost of labor is more cyclical than the average wage (Kudlyak, 2014; Basu and House, 2016). Third, the wage of new hires is a cyclical as the wage of incumbent worker (Gertler, Huckfeldt, and Trigari, 2020; Grigsby, Hurst, and Yildirmaz, 2021).

We present results from a simulated panel of entrepreneurs and workers in our model. We estimate equation (67) using different measures of wages on the left-hand side.

$$\log w_{ijt} = \beta_0 + \beta_1 (Recession_t = 1) + u_{ijt} \tag{67}$$

We construct a measure of the risk-neutral user cost of labor similar to the one proposed by Kudlyak (2014); Basu and House (2016). We call this the *risk-neutral* user cost of labor as it does not reflect the true user cost of labor in our model, where the present discounted value of wages should be discounted with the entrepreneurs' specific stochastic discount factor. We define the risk-neutral user cost of labor UC_t^{RN} in equation (68) as the difference between the average present discounted value of wages for a match created

	Wage	New hire wage	User cost
	(1)	(2)	(3)
$Recession_t = 1$	0.003	0.003	0.008

Table 15: Estimates of equation (68) using different measures of log-wage on the left-hand side. Estimates are obtained from a simulated panel of 20,000 entrepreneurs for 2000 periods (500 years).

at t minus the average present discounted value of wages for a match created at $t+1$.

$$UC_t^{RN} = E_t[PDV_{jt}^{RN}] - \beta(1-\phi)E_{t+1}[PDV_{jt+1}^{RN}] \tag{68}$$

To construct a measure of this present discounted value PDV_{jt}^{RN} for each firm j we use the present discounted value of all wages paid over the employment relationship discounted using $\beta(1-\phi)$ as a discount factor and truncating the summation after seven years as in Kudlyak (2014).

We report estimates of equation (67) in Table 15. The average wage falls by only 0.3% in an average recession, meaning that the average wage is only mildly procyclical in our model. Moreover, by comparing Columns 1 and 2 the average wage of new hires is a cyclical as the average wage of incumbents workers (Gertler, Huckfeldt, and Trigari, 2020; Grigsby, Hurst, and Yildirmaz, 2021). Finally, we find that drop in the average risk-neutral user cost of labor is almost three times larger then the drop in the average wage, consistent with evidence that the user cost of labor is substantially more cyclical than the average wage (Kudlyak, 2014; Basu and House, 2016).

D Model extensions

D.1 Limited commitment

One feature of the baseline model presented in Section 2 is that both entrepreneurs and workers can commit to a wage contract upon matching. Firm commitment is a common assumption in the literature studying dynamic wage contracts, as in Harris and Holmstrom (1982), Balke and Lamadon (2022), and is often motivated by firms' reputational concerns. Moreover, this assumption suits well a country like Italy where firing workers is particularly costly.

We propose an extension of the model presented in Section 2 where the contract is subject to limited commitment by workers. In particular, workers cannot commit to stay in an employment relationship if their value of being matched falls below the value of non-employment. In principle, when workers cannot commit to a wage contract, firms may implicitly fire them by offering a continuation value that is below the value of non-employment. We explicitly rule out this case, assuming that separation is exogenous. In practice, if firms had to pay a high cost Φ upon separation, it would never be optimal for them to offer a continuation value that triggers separation (Souchier, 2023). As a result, the optimal contract must satisfy an additional incentive compatibility constraint, namely that the continuation value of workers is always greater than the value of non-employment.

The problem of a matched entrepreneur in the economy with limited commitment is defined recursively in (69). As illustrated in Section 2, the value of being matched with a worker at the end of the period is equal to the flow utility of consumption plus the expected continuation values of the entrepreneur. With probability $(1-\phi)$, the match will survive until the end of the next period, while with probability ϕ the match will separate and the entrepreneur will get the continuation value V of being vacant at the end of next period. The optimal wage contract, which is defined recursively by state-contingent wages $w'(z',\xi')$ and state-contingent continuation values $W'(z',\xi')$ must satisfy a promise keeping constraint and an incentive compatibility constraint. To simplify the notation we suppress the dependence of $\mathcal{U}$ on the aggregate state S, as according to Proposition 1 the value $\mathcal{U}$ is constant over time.

$$
\begin{aligned}
J(m, W, z, S) = \max_{\substack{c^e, b', k', m'(z',\xi'),\\ w'(z',\xi'), W'(z',\xi')}} \Bigg\{ & v(c^e) + \beta(1-\phi) \underbrace{\mathbb{E}\left[J(m'(z',\xi'), W'(z',\xi'), z', S')|z, S\right]}_{\text{not separate}} \\
& + \beta\phi \underbrace{\mathbb{E}\left[V(m'(z',\xi'), z', S')|z, S\right]}_{\text{separate}} \Bigg\}
\end{aligned} \tag{69}
$$

$$\text{(Budget constraint : } \lambda^e) \qquad c^e + k' \leq m + qb'$$

$$\text{(Net worth : } \eta(z',\xi')) \qquad m'(z',\xi') \leq z'f(k') + (1-\delta)k' - w'(z',\xi') - b'$$

$$\text{(Collateral constraint : } \mu) \quad b' \leq \xi k'$$

$$\text{(Promise keeping : } \gamma) \qquad W \leq \mathbb{E}\Big[u(w'(z',\xi')) + \beta(1-\phi)W'(z',\xi') + \beta\phi\mathcal{U}|z,S\Big]$$

$$\text{(IC-worker : } \varrho(z',\xi')) \qquad \beta(1-\phi)\mathcal{U} \leq \beta(1-\phi)W'(z',\xi')$$

In the presence of limited commitment, the optimal wage contract does not imply perfect risk-sharing between the entrepreneur and the worker. Indeed, while the optimality condition for the state-contingent wages is unchanged as in (70), the optimality condition for the state-contingent continuation value leads to equation (71). This equation implies that the multiplier on the promise-keeping constraint γ is not constant over the length of a match if at some point in the future the incentive compatibility constraint will bind. Intuitively, perfect risk-sharing fails whenever the entrepreneur has to deviate from the optimal contract presented in Section 3 in order to guarantee the worker a continuation value at least greater than $\mathcal{U}$.

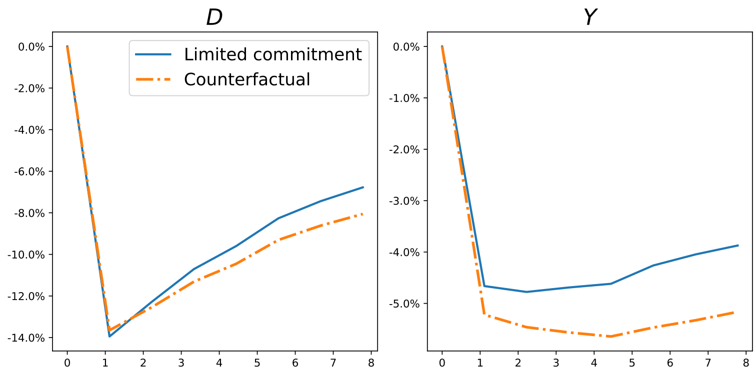
$$\eta(z',\xi') = \gamma w'(z',\xi') \tag{70}$$

$$\gamma + \varrho(z',\xi') = \frac{\partial J(m'(z',\xi'), W'(z',\xi'), z', S')}{\partial W'(z',\xi')} \tag{71}$$

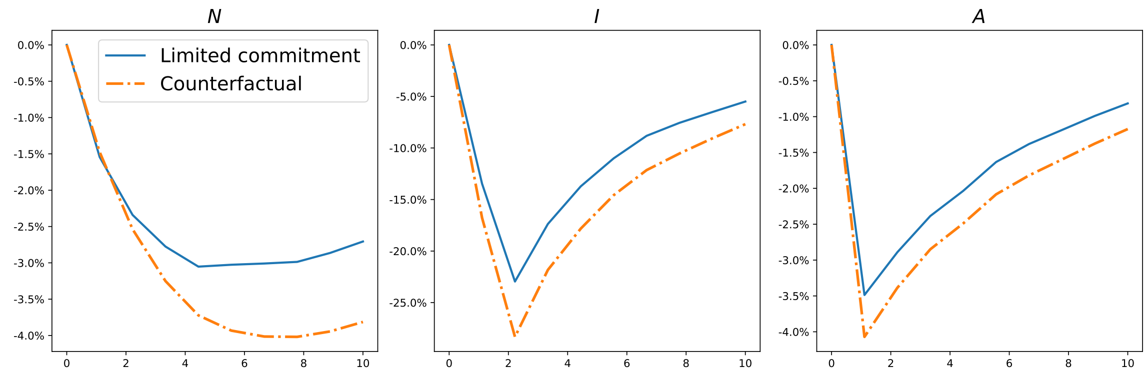
Note that the incentive compatibility constraint binds whenever the entrepreneur would like to offer the worker a continuation value that is too low. In practice, there can be two cases in which the IC constraint binds. First, when the entrepreneur wants to *frontload* wages by paying workers a high wage today and lower wages in the future. Second, when the entrepreneur would offer a state-contingent continuation value $W'(z',\xi')$ for some z',ξ' that is below $\mathcal{U}$. Crucially, none of these cases impairs firms' ability to temporarily backload wages when the financial constraint binds. In fact, when firms temporarily backload wages, they offer a *high* continuation value to the worker, as they would like to pay relatively higher wages in the future than today. Intuitively, when firms temporarily backload wages to ease the effects of financial constraints, they also implicitly provide workers incentives to stay.

In the next section, we show that the main quantitative results presented in Section 5 extend to the economy with limited commitment.

D.1.1 Quantitative analysis: limited commitment



(a) Aggregate debt and output



(b) Aggregate employment, investment, and productivity

Figure 9: Impulse response functions for aggregate debt (D), output (Y), employment (N), investment (I) and productivity (A) in response to an aggregate financial shock. The solid line plots impulse response functions in the economy with limited commitment by workers, and the dashed line plots impulse response functions in the counterfactual economy with no commitment. We compute $2 \times M$ simulations of length T. We draw M sequences of uniform random numbers that we use to simulate realizations of ξ. In the first M simulations we set $\xi = \xi_L$ at $T-10$. The IRFs are computed taking the difference in logs between the first and second set of simulations from $T-10$ to T, averaging across M.

We calibrate the model with limited commitment following the same strategy described in Section 5. Then, we compare the calibrated model with limited commitment to the same counterfactual economy discussed in Section 5. Figure 9 plots the impulse response functions to a drop in ξ in the calibrated model with limited commitment and in the counterfactual economy where firms cannot commit to future wages (i.e., firms cannot temporarily backload wages, nor they can adjust wages over time and in response to shocks). As usual, we recalibrated the value of ξ_L in the counterfactual economy to obtain the same drop in aggregate debt. Although the two economies experience the same drop in aggregate debt, output decreases more in the economy with limited commitment. The results are qualitatively similar to those presented in Figure 2 from Section 5: for the same drop in corporate debt output falls more in response to an aggregate financial shock when we limit firms' ability to backload wage payments and adjust wages in response to shocks.

D.2 Epstein-Zin Preferences

The baseline model presented in Section 2 assumes that workers have standard CRRA preferences, and in Section 3 we show that the key properties of the optimal wage contracts between entrepreneurs and workers depend on the ratio between their relative risk aversion coefficients. As it is well known, with CRRA preferences the relative risk aversion coefficient is also the inverse of the elasticity of intertemporal substitution (EIS). In principle, both the risk-aversion coefficient and the EIS of workers should matter. If workers have a large coefficient of RRA, we should expect them to be less inclined to accept wage contracts that are sensitive to idiosyncratic shocks. On the other hand, if workers have small EIS, we should expect them to be less inclined to accept wage payments that vary over time. In this section we derive a version of the baseline model where workers have Epstein-Zin preferences, that we use to better highlight the role played by risk and intertemporal substitution. In order to simplify the exposition we illustrate the model when Proposition 1 holds.

The value of a matched worker with promised utility W at the end of period (i.e., in the afternoon) is defined recursively as

$$W \leq E\left[\left\{(1-\beta)w'(z',\xi')^{1-\rho} + \beta\left[(1-\phi)W'(z',\xi') + \phi\mathcal{U}^{1-RRA}\right]^{\frac{1-\rho}{1-RRA}}\right\}^{\frac{1-RRA}{1-\rho}}\right]$$

Note that this definition is equivalent to define the value of a matched worker with promised utility W in the morning, *before* wages are paid, as

$$W \leq \left\{(1-\beta)w^{1-\rho} + \beta E_t\left[(1-\phi)W'\left(z',S'\right)^{1-RRA} + \phi\mathcal{U}^{1-RRA}\right]^{\frac{1-\rho}{1-RRA}}\right\}^{\frac{1}{1-\rho}}$$

that is more similar to the standard timing used with these preferences.
The problem of matched entrepreneurs in the afternoon is identical to (2), but with a modified version of the promise keeping constraint.

$$\begin{aligned} J(m,W,z,S) = \max_{\substack{c^m,b',k',m'(z',\xi'),\\ w'(z',\xi'),W'(z',\xi')}} \Bigg\{ & v(c^m) + \beta(1-\phi)\underbrace{\mathbb{E}\left[J(m'(z',\xi'),W'(z',\xi'),z',S')|z,S\right]}_{\text{not separate}} \\ & + \beta\phi\underbrace{\mathbb{E}\left[V(m'(z',\xi'),z',S')|z,S\right]}_{\text{separate}}\Bigg\} \end{aligned}$$

$$
\begin{aligned}
&(\text{Budget constraint} : \lambda^e) && c^m + k' \leq m + qb' \\
&(\text{Net worth} : \eta(z',\xi')) && m'(z',\xi') \leq y(k',z') + (1-\delta)k' - w'(z',\xi') - b' \\
&(\text{Collateral constraint} : \mu) && b' \leq \xi(S)k' \\
&(\text{Promise keeping} : \gamma) && W \leq \\
& && E\Big[\Big\{(1-\beta)w'(z',\xi')^{1-\rho} + \beta\left[(1-\phi)W'(z',\xi') + \phi\mathcal{U}^{1-RRA}\right]^{\frac{1-\rho}{1-RRA}}\Big\}^{\frac{1-RRA}{1-\rho}}\Big]
\end{aligned}
$$

The optimality conditions for state-contingent wages and promised values are

$$
\begin{aligned}
\eta(z',\xi') =&\gamma\Big\{(1-\beta)w'(z',\xi')^{1-\rho} + \beta\left[(1-\phi)W'(z',\xi') + \phi\mathcal{U}^{1-RRA}\right]^{\frac{1-\rho}{1-RRA}}\Big\}^{\frac{\rho-RRA}{1-\rho}} \\
&(1-RRA)(1-\beta)w'(z',\xi')^{-\rho} \\
\gamma' =&\gamma\Big\{(1-\beta)w'(z',\xi')^{1-\rho} + \beta\left[(1-\phi)W'(z',\xi') + \phi\mathcal{U}^{1-RRA}\right]^{\frac{1-\rho}{1-RRA}}\Big\}^{\frac{\rho-RRA}{1-\rho}} \\
&\times\left[(1-\phi)W'(z',\xi') + \phi\mathcal{U}^{1-RRA}\right]^{\frac{RRA-\rho}{1-RRA}}
\end{aligned}
$$

Conceptually these optimality conditions are similar to the simple risk-sharing condition from Section 3, where now the marginal value of a dollar for a worker earning $w'(z',\xi')$ does not depend only on the current wage payment, but also on future promised utilities. In order to highlight the role played here by the EIS and the RRA coefficient, it is useful to consider three illustrative examples.

Example 1: static problem. In order to highlight the role of the RRA coefficient, consider a two period economy where agents contract in the first period the wages will be paid in the second period. We have that for any pair (z_1',ξ_1'), (z_2',ξ_2'):

$$
\frac{\eta(z_1',\xi_1')}{\eta(z_2',\xi_2')} = \frac{w'(z_1',\xi_1')^{-RRA}}{w'(z_2',\xi_2')^{-RRA}}
$$

In this setting wages solve a pure infra-temporal risk-sharing problem, as there is no dynamic, that depends solely on the risk aversion coefficient of workers RRA, and on the risk aversion of entrepreneurs through the usual multiplier η.

Example 2: deterministic problem with no separation. In order to highlight the role of the EIS, consider an economy with no idiosyncratic and aggregate risk, so that z and S are constant and $\phi = 1$. We can re-arrange the optimality conditions to obtain

$$
\frac{\eta_t}{\eta_{t+1}} = \frac{w_t^{-\rho}}{w_{t+1}^{-\rho}}
$$

In this setting wages solve a pure inter-temporal problem, as there are no shocks, that depends solely on the EIS coefficient of workers ρ, and on the EIS of entrepreneurs through the usual multiplier η.

Example 3: deterministic problem with separation. In order to highlight the role of separation, consider an economy where z and S are constant, but matches are subject to the idiosyncratic risk of separation. The optimality conditions for wages read:

$$\frac{\eta_t}{\eta_{t+1}} = \left[\frac{(1-\phi)W' + \phi\mathcal{U}^{1-RRA}}{W'}\right]^{\frac{\rho-RRA}{1-RRA}} \frac{w_t^{-\rho}}{w_{t+1}^{-\rho}}$$

Note that whenever workers would prefer to be employed, we have that $W' < \mathcal{U}^{1-RRA}$. Then we have that

$$\frac{\eta_t}{\eta_{t+1}} > \frac{w_t^{-\rho}}{w_{t+1}^{-\rho}} \quad \Leftrightarrow \quad RRA > \rho$$

which means workers wages are less backloaded than they would in the CRRA case if $RRA > \rho$. Intuitively, when $RRA > \rho$ workers have a preference for early resolution of uncertainty, where here uncertainty comes from separation occurring with probability ϕ. Thus, workers are less willing to accept backloaded wages as there is a positive probability ϕ that they won't get the future promised utility W'.